Other-regarding Reference Points: A Matter of Perspective*

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April 12, 2018

Abstract

We combine empathetic and reference-dependent preferences to a more comprehensive model of decision-making. Empathy can refer to other people’s basic needs (exogenous reference points) as well as others’ ambitions in the form of social comparisons. Empathy with others’ ambitions results in inequality aversion (Fehr and Schmidt 1999, QJE, Vol. 114, pp. 817-868). Here our model also implies that the Fehr-Schmidt parameters are positively correlated. In addition our model accommodates social value orientations, showing that a “competitive” orientation implies sensitivity to own ambitions. For linear public goods games, we furthermore provide an explanation for “hump-shaped” preferences which assumes sensitivity to others’ needs.

Keywords: distributive fairness, empathy, inequality aversion, linear public good, loss aversion, reference dependence, social value orientation

JEL Codes: D63, D81, H41

*This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. I am indebted to Claudia Niemeyer for drawing my attention to experimental observations concerning hump-shaped preferences in linear public goods games.

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1 Introduction

Why do people follow fairness principles? Even though well-known economic models of other-regarding preferences such as Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) are quite consistent with experimental observations, these theories nevertheless fail to answer this simple question (e.g., Bergh, 2008). People are just assumed to have these preferences, as this assumption best accommodates experimental observations.

Motivated by this and other criticism raised by Bergh (2008) against inequality aversion, we present a more comprehensive model of preferences for distributive fairness. This model combines reference-dependent preferences from prospect theory (e.g., Köszegi and Rabin, 2006; Santos-Pinto et al., 2015; Tversky and Kahneman, 1991, 1992) with empathetic preferences as described for example by Binmore (1994, Chapter 4.3.1). Both of these approaches are grounded on well-established preference foundations which accordingly also provide a solid basis for our arguments. Moreover, both approaches have evolutionary roots which permit the elicitation of preferences through physiological measurements, including oxytocin for empathy (e.g., Rodrigues et al., 2009; Zak et al., 2007) and dopamine for subjective well-being related to “expectations” (Rutledge et al., 2014, 2016). Such independent, non-strategic measurements of model parameters avoid a “serious concern” (Bergh, 2008 p. 1791) with inequality aversion that the Fehr-Schmidt parameters are often calibrated ex post by means of the same strategic interaction which the models then intends to explain.

In addition to the exogenous reference points associated with loss aversion by Köszegi and Rabin (2006), our model also acknowledges social comparisons as a form of reference dependence. From the perspective of the decision-maker (DM), these two components represent DM’s needs, specified by an exogenous payoff target, and her ambitions with respect to social status, specified by the payoffs of other people she compares herself to. In both cases DM enjoys a gain utility, if her outcome exceeds the target, and suffers a loss utility, if her outcome falls short. Furthermore, if DM assumes the perspective of another person, she also experiences a gain-loss utility proportional to the same gain or loss that other person (presumably) experiences. Empathy with others’ needs corresponds to a gain-loss utility with respect to exogenous reference points. It is therefore conceptually similar to loss aversion. Empathy with others’ ambitions accounts for a gain-loss utility from social comparisons. Accordingly, DM follows fairness principles, because she empathizes with others’ needs and ambitions.

We argue that each of the four components of this ENA model (“Empathy with Needs and Ambitions”) – i.e., sensitivity to own and others’ needs as well as to own and others’

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1 See especially Harman (2010) and the references cited there for evidence of empathy in animals. Kropotkin (1902) recounts much anecdotal evidence for “mutual aid” in animals and early human societies. Bester and Güth (1998) provide theoretical arguments for the evolutionary roots of altruistic preferences. The evolutionary roots of reference-dependent preferences are less well-established. However, loss aversion has been documented in capuchin monkeys (Lakshminarayanan et al., 2011), indicating that this might be a genetic trait at least in higher-developed mammals.

2 Fehr and Schmidt (1999) are more careful in this respect by using the ultimatum game (Güth et al., 1982) to calibrate the parameters before applying their model to other games. However, it remains unclear why ultimatum bargaining (a strategic interaction of two players) is a good baseline scenario, e.g. when investigating an n-player public goods game (a strategic interaction of multiple players) or a dictator game (a non-strategic interaction of two players).
ambitions – is necessary within the framework of prospect theory to accommodate certain experimental findings. This is obviously true for loss aversion, representing sensitivity to own needs. Sensitivity to own and others’ ambitions are the core components of inequality aversion (Fehr and Schmidt, 1999). However, the necessity of sensitivity to own ambitions is best demonstrated by observations of “competitive” social value orientations (e.g., Chen and Fischbacher, 2016; Liebrand and McClintock, 1988; Murphy et al., 2011) in a non-strategic allocation task. We show theoretically that, within our model, such orientations are otherwise impossible. Regarding the relevance of empathy with others’ ambitions we cite Loewenstein et al. (1989), who observe that the gain utility from earning more than another person varies with the context. Specifically, the authors observe a utility function in accordance with prospect theory (increasing, kink at zero) for business interactions, but a function in accordance with inequality aversion (maximum at zero) for interactions among friends. Our model can accommodate both contexts, but only if DM is sensitive to others’ ambitions. Empathy with others’ needs, finally, receives unexpected support in the form of “hump-shaped” preferences for public good provision elicited through the strategy method (Fischbacher et al., 2001). Our model suggests, that this behavior is not only purposeful, but represents an intermediate type between selfish players and conditional cooperators: DM conditionally cooperates, but only until the rest of the group has earned enough to satisfy their exogenous reference points. In proving these results, we also show that the ENA model can accommodate the models by Fehr and Schmidt (1999) and Charness and Rabin (2002).

The context-dependence of our model’s sensitivity parameters contrasts with most other fairness models, which usually assume that preferences are independent of context. The ENA model can accommodate context effects on other-regarding preferences such as social identity (e.g., Akerlof and Kranton, 2000; Chen and Li, 2009; Loewenstein et al., 1989), group size (e.g., Stahl and Haruvy, 2006), past behavior and intentions of others (e.g., Charness and Rabin, 2002; Falk and Fischbacher, 2006), and mood (e.g., George, 1991; Rutledge et al., 2016; Hanley et al., 2017). However, except for group size, we will leave the formal analysis of such context effects for future work.

The rest of the paper proceeds as follows. After discussing additional related literature in Section 2 and giving a short introduction to reference-dependent preferences in Section 3, we present our theoretical model in Section 4. Subsequent sections are concerned with model applications, including inequality aversion (Section 5), social value orientations (Section 6), and public good provision (Section 7). Section 8 concludes.

2 Related literature

Empirical research leaves no doubt that many decision-makers consider the payoff of another person involved in a similar situation as a reference point. Examples include the seminal studies by Güth et al. (1982) on ultimatum bargaining or Forsythe et al. (1994) on the dictator game. Yet, despite the acknowledgement of this reference point, attempts to reduce these findings to prospect theory have so far been unsuccessful. For example, Tavoni (2010) goes as far as to introduce a “distributive reference point”, but does not pursue this parallel to prospect theory any further. Loewenstein et al. (1989) demonstrate the probably most important obstacle to this endeavor: While subjects usually demonstrate an aversion to
earning less than others, which is consistent with prospect theory, they are often similarly aver}
ese to earning more than others, which seems to contradict this theory. Some empirical researchers also reject alternative explanations for their findings that rely on prospect theory, because it is impossible to know after the fact what reference point may have influenced a subject’s decision (see, e.g. Sonnemans et al., 1998). This view is strengthened by Rohde (2010) and Saito (2013), who derive preference foundations for inequality aversion, independently of prospect theory. Similarly, Kenschbamer (2015) shows that most economic fairness models based on social comparisons satisfy the same monotonicity assumptions, but fails to draw a connection between social comparisons and loss aversion. Clark et al. (2008) provide empirical evidence in support of such a connection. However, they only consider gain-loss utilities from the perspective of the decision-maker, disregarding the possibility of empathy with others. Andreoni (1990), by contrast, models the “warm glow” that results from helping others (presumably to satisfy their needs), but ignores the impact of social comparisons on this decision. Oxbury (2004) argues that a lack of social status causes cognitive dissonance. While he acknowledges a relation to prospect theory, his model neither incorporates loss aversion, nor empathy.

The idea of empathetic preferences traces back to the early days of economics (Smith, 1759). Under the influence of Von Neumann and Morgenstern (2007), many modern economists may consider empathy as a contradiction to the profit-oriented, self-centered homo economicus. However, several authors such as Harman (2010, p.359) have recently rediscovered this idea. Binmore (1994, 1998) makes a strong case for the evolutionary roots of empathetic preferences. This argument is crucial for justifying interpersonal comparisons in his conception of a social contract. Studies on oxytocin (e.g. Rodrigues et al., 2009; Zak et al., 2007) give empirical evidence in support of this contention.

Another empirical study, Rutledge et al. (2016), connects subjective well-being with expectations related to decisions under risk and social comparisons. Although their empirical model is structurally similar to our theoretical one in that it combines multiple reference points to calculate a single utility value (a hedonic utility in their case), it ignores important concepts such as empathy and context dependence. Nevertheless, this approach provides an important benchmark for our own theoretical investigation. For instance, Rutledge et al. (2016) do not observe any correlation between the guilt and envy parameters of the Fehr-Schmidt model. These findings therefore justify our more general approach to combine ambitions and needs in one model.

3See also Fudenberg and Levine (2012).
4In fact, welfare economics in general at least implicitly assumes that the social planner, as a theoretical construct, is able to empathize with the preferences of individuals. I am indebted to Thomas Aronsson for making this observation.
5Caplin and Dean (2008) provide behavioral foundations for this approach (see also Rutledge et al. 2010), which is based on physiological measurements of dopamine release and blood oxygen levels (Rutledge et al., 2014).
6This is particularly problematic, because Rutledge et al. (2016) analyze repeated decisions in which earnings are cumulative, possibly causing an income effect in later rounds, which might affect the context in which their “social trials” occur.
7It is possible that this is actually a reaction to the computer agent used in the experiments “social trials”. Although programmed with “typical economic preferences” (i.e. risk and loss aversion, ibid. p.6), the computer would not react to actual developments in the experiment like a “normal” person might.
3 Reference-dependent preferences as a multiattribute utility function

The following theoretical model is based on Köszegi and Rabin [2006] in connection with an “attribute-specific evaluation” of multiattribute prospects [Bleichrodt et al., 2009]. We assume that the decision-maker (DM), denoted by the subscript $i$, belongs to a group of $n$ persons. DM chooses from a set $X \subset \mathbb{R}^n$ comprising outcomes $x = (x_1, \ldots, x_i, \ldots, x_n)$. Each outcome $x \in X$ denotes the monetary payoff received by each person in the group; DM receives $x_i$.

Although prospect theory has been developed for decision-making under risk or uncertainty, models of other-regarding preferences commonly involve decisions under certainty. Accordingly, all applications of our model in this paper assume that there are no different states of nature that would result in different outcomes for the same prospect.\footnote{This corresponds to the approach taken in Tversky and Kahneman [1991].} We therefore identify each outcome $x$ with the constant prospect that generates it. As a side-effect of this assumption, we do not need to specify probability weighting functions and can concentrate solely on reference-dependent preferences. However, this assumption does not rule out a generalization of our model to other-regarding preferences under risk or uncertainty.

DM has preferences over $X$ which can be represented by the multiattribute utility function $u_i(x|r) : X \to \mathbb{R}$. $u_i(x|r)$ aggregates DM’s consumption utility $v_C(x_i)$ with gain-loss utilities generated by $M$ comparisons of attribute values $v_K(x) : X \to \mathbb{R}, 1 \leq K \leq M$, each associated with a reference point $r_K \in \mathbb{R}$. As will be explained below in more detail, each attribute value $v_K(x)$ represents the payoff of an individual person in a comparison to that person’s reference payoff $r_K$. This reference point may be exogenous or another person’s payoff. A person $j$’s payoff $x_j$ can be used for different comparisons, if multiple reference points are associated with this person. $r = (r_1, \ldots, r_M)$ denotes all $M$ reference points that DM perceives as relevant to her decision, hers and possibly those of other persons. The attribute-specific evaluation [Bleichrodt et al., 2009] consists in the assumption that each of these reference points induces a separate gain-loss utility $\mu_i(v_K(x)|r_K)$. The properties of $\mu_i(\cdot) : \mathbb{R} \to \mathbb{R}$ are consistent with the behavioral foundations laid out by prospect theory (cf. Köszegi and Rabin [2006])

\begin{itemize}
  \item[(A0)] $\mu_i(x|r)$ is continuous in $x$ for all $x, r \in \mathbb{R}$ and twice differentiable with respect to $x$ at $x \neq r$. Furthermore $\mu_i(r|r) = 0$.
  \item[(A1)] $\mu_i(x|r)$ is strictly increasing in $x$.
  \item[(A2)] If $y > x > r$, then $\mu_i(y|r) + \mu_i(r|y) < \mu_i(x|r) + \mu_i(r|x)$.
  \item[(A3)] If $x > r$, then $\mu_i''(x|r) \leq 0$. If $x < r$, then $\mu_i''(x|r) \geq 0$.
\end{itemize}

\footnote{Köszegi and Rabin [2006] define the assumptions only for $r = 0$, which is too restrictive when considering multiple reference points. However, except for Assumption A2, the generalization is straightforward. Our version of A2 retains the property of typical parametric utility functions that, if the decision maker is loss neutral ($\lambda_i = 1$) and $\mu_i(x|r)$ has the same absolute curvature $|\mu_i''(x|r)|$ for both gains and losses, $\mu_i(x|r)$ is point-symmetric at $x = r$.}
(A4) \( \lambda_i \equiv \frac{\mu_{i,-}(r)}{\mu_{i,+}(r)} \geq 1 \), with \( \mu_{i,+}(r) \equiv \lim_{x \to r} \mu_{i}'(x | r) \), \( \mu_{i,-}(r) \equiv \lim_{x \to r} \mu_{i}'(-x | r) \).

Here \( \lambda_i \) denotes DM’s the extent of loss aversion. We assume that \( \mu_i(\cdot) \) applies to all reference points in the vector \( r \). Accordingly, we treat DM’s risk and loss preferences as context-independent for the purpose of this analysis (see also Section 4.1).

As we only consider decisions under certainty, we restrict our analysis to (piece-wise) linear utility functions, letting \( u_C(x_i) = x_i \) and replacing A3 with the following (cf. Köszegi and Rabin 2006):

\[ u_i(x|r) = x_i + \sum_{K=1}^{M} \eta_{i,K} \mu_i(v_K|x) r_K \]  

(A3’) If \( x \neq r \), then \( \mu_i''(x|r) = 0 \).

The models of other-regarding preferences we consider, especially inequality aversion and social value orientation, assume that \( u_i(x|r) \) is additively separable into the utilities of its attributes:\(^{10}\)

\[ u_i(x|r) = x_i + \sum_{K=1}^{M} \eta_{i,K} \mu_i(v_K|x) r_K \]  

Here \( \eta_{i,K} \geq 0 \) is the weight that DM places on utility component \( K \), i.e. her sensitivity to gains and losses with respect to the comparison of the attribute value \( v_K(x) \) to the reference point \( r_K \). The weight of the consumption utility \( x_i \) is normalized to 1.\(^{11}\) In contrast with \( \mu_i(\cdot) \), the \( \eta_{i,K} \) are assumed to be context-dependent and must therefore be elicited anew in each experimental investigation.

As the attribute values \( v_K(x) \) are not necessarily independent of each other,\(^{12}\) we also make the following monotonicity assumptions for \( u_i(x|r) \):

(M) For any \( x \equiv (x_i, x_{-i}) \in X \) and \( \delta > 0 \): \( u_i((x_i + \delta, x_{-i})|r) > u_i(x|r) \).

(E) For any \( x \in X \) and \( \Delta \equiv (\delta, \ldots, \delta) \in \mathbb{R}^{n} \), \( \delta > 0 \), such that \( x + \Delta \in X \):
\[ u_i(x + \Delta|r) > u_i(x|r) \]

Assumption M corresponds to “strict m-monotonicity” in Kershbamer (2015) and establishes that DM prefers to increase her consumption no matter her reference points. Assumption E, which is similar to “strict equal-material-payoff-monotonicity” in Kershbamer (2015), implies a preference for more efficient outcomes (Pareto improvements) as long as payoff differences are unchanged.\(^{13}\) Assumption M, although seemingly innocuous as a rationality requirement, can nevertheless severely limit other-regarding preferences, as will be demonstrated in Section 4 on social value orientation.

\(^{10}\) Köszegi and Rabin (2006) make the same assumption, but each of their attributes has exactly one reference point.

\(^{11}\) Alternatively, define \( \eta_{i,C} \) as the weight DM places on \( x_i \) and normalize the weights as probabilities, so that \( \eta_{i,C} + \sum_{K=1}^{M} \eta_{i,K} = 1 \). This assumes, of course, that the researcher is aware of the entire set of relevant reference points, which is unlikely.

\(^{12}\) For example, Engelmann and Strobel (2004) make this point in their empirical comparison of different fairness models.

\(^{13}\) Kershbamer (2015) makes two additional assumptions in his characterization of models of distributive fairness. The first, combining completeness, transitivity, and continuity of preferences, is implied by our assumption A0 combined with the additive separability of \( u_i(x|r) \). The second, “piecewise o-monotonicity”, is only satisfied by our model if reference points are restricted to social comparisons (i.e. gain-loss utility from payoff differences).
4 Other-regarding reference-dependent preferences

In accordance with the previous section, we can choose reference points \( r \) to reflect other-regarding preferences. We distinguish between self-centered references, in which DM compares her own payoff \( x_i \) to a reference point, and other-centered references, in which DM uses another person’s payoff \( x_k \) for the comparison. Reference points fall into two categories:

- **Needs (N):** Each person \( k \) has an exogenous reference point \( r_k \) representing the person’s basic needs.
- **Ambitions (A):** Each person’s payoff \( x_k \) represents that person’s social status. The person’s ambitions correspond to the social comparisons with the status of other persons.\(^{14}\)

The ENA-model (for “empathy with others’ needs and ambitions”) includes needs and ambitions in the same multiattribute utility function for DM in a weighted sum of all gain-loss utilities from different reference points in addition to the consumption utility \( x_i \):\(^{15}\)

\[
U_{i,ENA}(x|r) = x_i + \eta_{i,N}^{j} \mu_i(x_i|r_j) + \eta_{i,A}^{j} \sum_{j \neq i} \mu_i(x_i|x_j) + \sum_{j \neq i} \eta_{i,N}^{j} \mu_i(x_j|r_j) + \sum_{j \neq i} \eta_{i,A}^{j} \mu_i(x_j|x_i) \tag{4.1}
\]

Here, the weight \( \eta_{i,R}^{j} \geq 0, R \in \{N,A\} \), denotes DM’s sensitivity to a particular reference point evaluated from the perspective of person \( j \). \( N \) and \( A \) refer to needs and ambitions, respectively. Self-centered references have the superscript \( i \), e.g. \( \eta_{i,N}^{i} \) for DM’s own needs (i.e. loss aversion). Essentially, \( \eta_{i,R}^{j} \) is a scaling factor for the gain-loss utility from a particular reference point. If DM is insensitive to the reference point, then \( \eta_{i,R}^{j} = 0 \).

Note that we have restricted social comparisons (i.e. ambitions) to either those of DM’s status (represented by \( x_i \)) compared to that of other persons status (represented by \( x_j \)) or those of another person’s status compared to that of DM. The former type of comparison is always weighted by \( \eta_{i,A}^{j} \) as it takes place from DM’s perspective. In the latter case, the weight \( \eta_{i,A}^{j} \) depends on the person with which DM empathizes. In the following analysis it is not necessary to consider social comparisons in which the decision-maker is not involved. Such comparisons are permitted by the model, however.

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\(^{14}\)A comparison to one’s own social status results in a gain-loss utility of zero, because \( \mu_i(r|r) = 0 \) for all \( r \).

\(^{15}\)We do not consider an other-centered version of this consumption utility according to which DM might attribute an absolute utility to the monetary payoff of other players. In our understanding, empathy does not go so far as to make it possible to buy goods with other people’s money, at least not under normal circumstances. However, given sufficient empirical justification, \( x_j \) could be added as a “virtual” consumption utility to DM’s utility function, but should then be discounted by a corresponding sensitivity parameter \( \eta_{i,C}^{j} \). In hypothetical decisions, \( x_i \) can be similarly discounted or dropped entirely.
Proposition 4.1. $u_i^{ENA}(x|r)$ satisfies monotonicity assumption $E$.

Proof. This follows immediately from the monotonicity of $\mu(\cdot)$ (assumption A1). □

Proposition 4.2. Assuming A3’, the two following statements are equivalent (n denotes group size):

1. $1 + \eta_{i,N}^i + (n - 1)\eta_{i,A}^i > \lambda_i \sum_{j \neq i} \eta_{j,A}^j$
2. $u_i^{ENA}(x|r)$ satisfies monotonicity assumption $M$.

Proof. See appendix. □

Proposition 4.2 gives an upper bound for the sensitivity parameters $\eta_{i,A}^i$, which is the more difficult to satisfy the larger $\lambda_i$. A DM with a high extent of loss aversion $\lambda_i$ accordingly experiences a conflict between rational behavior (monotonicity) and empathy with others’ ambitions. However, although loss aversion may limit the sensitivity to others’ ambitions, this is not true for the sensitivity to others’ needs.

4.1 Context

We assume a different weight for each reference point, because it seems reasonable to expect that some references are more salient than others, depending on the context in which the decision takes place (e.g., the experimental framing). We expect in particular the empathy parameters $\eta_{i,N}^i$ and $\eta_{i,A}^i$ to be affected by the following context factors:

- **Social identity** (e.g. Akerlof and Kranton, 2000; Chen and Li, 2009; Loewenstein et al., 1989): DM empathizes with her family’s needs and ambitions more than with those of a business acquaintance, or treats women differently than men.

- **Group size** (e.g. Stahl and Haruvy, 2006): The more people are affected by DM’s decision, the less likely she knows them all well enough to empathize with their needs or ambitions.\(^{16}\)

- **Past behavior and intentions of others** (e.g. Falk and Fischbacher, 2006): The intention behind a previous action, i.e. whether it is kind or hostile, affects subsequent decisions by inducing a desire for reciprocity.

- **Mood** (e.g. George, 1991; Hanley et al., 2017; Rutledge et al., 2016);\(^{17}\) A good mood facilitates prosocial behavior. As past outcomes can affect the decision-maker’s mood, this is related to the preceding factor.

\(^{16}\)It is also more difficult to coordinate behavior in larger groups, which in turn can adversely affect cooperation (e.g. Felovich and Grossman, 2015). However, by itself this effect does not mean that empathy with others also decreases if more people are affected by a decision.

\(^{17}\)See also the additional references provided by Hanley et al. (2017).
Considering that prosocial decisions take longer than selfish ones (Chen and Fischbacher, 2016; Liebrand and McClintock, 1988), one might also expect mental fatigue or distractions to hinder DM’s ability to empathize with others. Each additional reference point factoring into a decision increases the amount of information required for an informed choice. Selfish decisions without any social comparisons are therefore easier to make than prosocial or competitive decisions (Liebrand and McClintock, 1988).

The experimental findings by Kahneman et al. (1986) and Konow (1996, 2000) suggest that context can also determine what is considered “fair” in a particular situation, i.e. set the precise value of the reference outcome. We believe that most of these effects (especially those concerning difference in effort, i.e. past behavior) are already accounted for by the above-mentioned factors. We therefore treat the reference outcomes $r$ as monetary payoffs (just like $x_i$ and $x_j$).

As mentioned above, we also assume that the extent of loss and risk preferences captured by $\mu_i(\cdot)$ is stable with respect to contextual differences, or at least more stable than the sensitivity parameters $\eta_{i,R}$. The main reason for this assumption is that it removes degrees of freedom from a model that already has many parameters. However, this assumption is an empirically testable hypothesis, which in fact is consistent with the observations by Dohmen et al. (2011) for risk attitudes in different contexts (e.g., career choices or financial matters). Yet, evidence to the contrary exists as well: Dohmen et al. (2017) report that risk aversion increases with age.

Füllbrunn and Luhan (2017) observe a lower extent of loss aversion if the outcome of a decision affects only a second person than if the same decision affects only the decision maker or both the decision maker and the second person. Chakravarty et al. (2011) report that subjects are more risk averse when deciding for themselves than in decisions on behalf of others. Benjamin et al. (2010) find that African American subjects who are made aware of their race are more risk averse compared to the control group. Similarly, intelligence is a (stable) genetic trait, which might affect the amount of information that can be processed with respect to sensitivity to reference points, independently of the context.

### 4.2 Measuring reference-dependent preferences

Reference-dependent preferences are part of the larger theoretical framework of prospect theory which provides methods to independently estimate the parameters of our model. A benchmark for $\mu_i(\cdot)$ can accordingly be determined in a neutral context via revealed preferences with respect to different prospects. Risk preferences in the gain domain are commonly determined via the decision task by Holt and Laury (2002). The extent of loss aversion $\lambda_i$ is estimated using “mixed gambles” (e.g. Santos-Pinto et al., 2015).

Once $\mu_i(\cdot)$ is known, it is possible to present DM with different prospects that also affect one or more additional persons’ payoffs. These prospects should be chosen in a way that

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18 Alternatively, the entitlement formula in Konow (1996) could be used as an additional reference point, possibly replacing the ambitions component.

19 Although the authors do not acknowledge the possibility that changing reference points drive this effect, they control for income, which certainly determines if one’s needs and ambitions can be satisfied.

20 A similar priming of gender has no effect, though.

21 Alternatively, probability weighting can be used to express risk attitudes. Compare in particular the concept of rank-dependent utilities described e.g. in Walker (2010).
DM is indifferent between them, unless she has other-regarding preferences. For example, say prospect $A$ corresponds to $x_i(A) = 10$ and $x_j(A) = 12$, prospect $B$ to $x_i(B) = 10$ and $x_j(B) = 20$, and prospect $C$ to $x_i(C) = 10$ and $x_j(C) = 1$. If $\eta^k_{i,R} = 0$ for all $k$ and $R$ (except possibly $\eta^j_{i,N}$), then DM is indifferent between $A$, $B$, and $C$, because she receives 10 in all cases. If DM prefers $B$ to $A$, then either $\eta^j_{i,N} > 0$, or $\eta^j_{i,A} > \eta^i_{i,A} \geq 0$, or both. In the first case, DM may feel a “warm glow” (Andreoni, 1990) from giving money to the other person. In the second case, DM instead empathizes with $j$’s ambitions. A combination of both effects is also possible. If DM prefers $C$ to $A$, she clearly places her own ambitions before the other person’s ambitions or needs, so that $\eta^i_{i,A} > 0$.

In more complex decisions, such as the ring measure used e.g. by Liebrand and McClintock (1988) for social value orientation (see Section 6), it may be helpful to test if DM’s extent of loss aversion $\lambda_i$ is correlated (positively or negatively) with her decision. For instance, Proposition 4.2 predicts that a high extent of loss aversion $\lambda_i$ is negatively correlated with the sensitivity to others’ ambitions $\eta^j_{i,A}$. If no such correlation is observed, it is unlikely that others’ ambitions are important in the investigated situation. This makes a more refined analysis of this utility component unnecessary.

However, because the sensitivity parameters are context-dependent (as explained above), they are difficult to measure precisely. This is especially true if the decision is influenced by multiple reference points. One option in this case is to equate sensitivity with attention to relevant information. For example, if DM wants to compare her payoff to that of another person $j$, she needs to access information about $x_j$. Eye tracking (e.g., Fiedler et al., 2013) and the analysis of mouse movements (Chen and Fischbacher, 2016) can reveal what kind of information is accessed by the subject, independently of the decision task. This in turn allows inferences regarding which reference points (if any) are relevant to this decision. A relative measure for $\eta^j_{i,R}$ can also be derived by eliciting $\mu_i(\cdot)$ not just in a neutral context, but also under a framing that stresses a particular reference point. This allows the conclusion that a specific reference point is more (or less) relevant than in a neutral context. The different vignettes used by Loewenstein et al. (1989) to vary social distance provide an example. Finally, assuming that deviations from a reference point cause cognitive dissonance (compare Konow 2000; Oxoby 2004), the sensitivity to a reference point should be proportional to the strength of a physiological reaction (stress or arousal) triggered whenever the subject processes relevant information. This is supported by the observations by Rutledge et al. (2016) that social comparisons affect subjective well-being.

5 Inequality aversion as prosocial behavior induced by empathy

In this section we elaborate on the importance of empathy for decisions affecting distributive fairness. For this purpose we assume for the moment that the effect of exogenous reference points is irrelevant to the decision, so that $\eta^j_{i,N} = 0$ for all $j$ (including $i$). We first show

\[\text{E.g., Etgen and Rosen (1993) link cognitive dissonance to an increased heart rate.}\]

\[\text{23This assumption is reasonable if 1) the payoff stakes are sufficiently high to satisfy all players’ needs and 2) the decision is not framed in a way that introduces new exogenous reference points (such as highlighting}\]
mathematically that the ENA-model can accommodate inequality aversion according to Fehr and Schmidt (1999). This derivation contrasts with earlier attempts (e.g. Rohde 2010; Saito, 2013) to directly reduce inequality aversion to preference axioms, as opposed to our (implied) indirect reduction to such axioms via prospect theory. These fairness preferences are context-dependent within our theoretical framework, in particular with respect to group size, and consistent with the empirical findings by Loewenstein et al. (1989). This is a novel result which follows neither from Fehr and Schmidt (1999) nor the other cited references (Rohde, 2010; Saito, 2013).

Fehr and Schmidt (1999) postulate that decision-makers are not only averse to earning less, but also more than others. Specifically, they define the following utility function for DM:

$$u_{iFS}^E(x) = x_i - \frac{\alpha_i}{n-1} \sum_{j \neq i} \max \{x_j - x_i, 0\} - \frac{\beta_i}{n-1} \sum_{j \neq i} \max \{x_i - x_j, 0\}$$

(5.1)

Fehr and Schmidt (1999) assume that $\alpha_i \geq \beta_i$ and $0 \leq \beta_i < 1$. These parameters are independent of group size $n$ and the person to which DM compares her payoff.

In order to bring the Fehr-Schmidt model in line with reference-dependent preferences, we must find a reference point that induces a negative gain-loss utility in DM whenever she earns more than another person $j$. Obviously, the reference point must concern the involved persons’ ambitions (A) to reflect a comparison of payoffs. And it cannot be self-centered, because then DM would enjoy a positive gain-loss utility whenever $x_i > x_j$. However, from the perspective of $j$, the same outcome is a loss with a negative gain-loss utility. So if DM empathizes with $j$’s ambitions, she suffers a disutility even though she earns more than $j$. Fehr and Schmidt (1999, p.824f.), in contrast, treat DM’s disutility as self-centered even if $x_i > x_j$.

By itself, this change of perspective is still insufficient for capturing inequality aversion, because $j$ conversely enjoys earning more than $i$ (see Figure 1 other-centered component). But a combination of self-centered and other-centered reference points (while excluding exogenous references) does the trick:

$$u_{iENA}^E(x|r) = x_i + \eta_{i,A}^j(n) \sum_{j \neq i} \mu_i(x_i|x_j) + \sum_{j \neq i} \eta_{i,A}^j(n) \mu_i(x_j|x_i)$$

(5.2)

Here, we treat group size $n$ as a factor that affects the context of the decision, so that the sensitivity parameters $\eta_{i,A}^j(n)$ are now functions of $n$. By contrast, the Fehr-Schmidt parameters are context-independent, so that in particular $\alpha_i(n) = \alpha_i$ and $\beta_i(n) = \beta_i$ for all $n$. We therefore let $\tilde{\eta}_{i,A}^j \geq 0$, for all $j$ (including $j = i$), denote sensitivity parameters that are also independent of group size. We have $\tilde{\eta}_{i,A}^j \equiv (n-1)\eta_{i,A}^j(n)$, for all $n$ and $j$ (including $j = i$). Accordingly, DM becomes less sensitive to individual payoff differences in larger

---

24We slightly modify the model to fit with our notation.

25If $n > 2$, Fehr and Schmidt (1999) also assume that DM does not care for payoff differences between other players.
Figure 1: Different components of inequality aversion in a decision with two persons $i$ and $j$. a) Gain-loss utility representing DM’s own ambitions (with $\eta_{i,A}^i = 1$ and $\lambda_i = 2.5$). b) Gain-loss utility representing DM’s empathy with others’ ambitions (with $\eta_{j,A}^i = 1$ and $\lambda_i = 2.5$). c) Aggregate utility $u^{ENA}_i(x|r)$ including both reference points and the consumption utility $x_i$. 
groups, which is plausible as there are more people to which she can compare herself.\footnote{Fehr and Schmidt (1999) normalize the gain-loss utility in a similar way “to make sure that the relative impact of inequality aversion on player i’s total payoff is independent of the number of players” (ibid., p.824).}

As $\alpha_i$ and $\beta_i$ are also independent of the reference person $j$, we further have $\eta_{i,A}^j(n) = \eta_{k,A}^j(n)$ for all $j \neq k$ except $j = i$.\footnote{This assumption is not satisfied if DM favors some group members over others, meaning that $u^{ENA}_i(x|r)$ is more general than $u^{FS}_i(x)$. We can conclude further, that $u^{FS}_i(x)$ should make the best predictions for a one-shot decision affecting a group with which DM has not previously interacted.}

We can now prove that the ENA-model can accommodate Fehr-Schmidt preferences:

**Theorem 5.1.** $u^{FS}_i(x)$ belongs to the class of utility functions specified by $u^{ENA}_i(x|r)$ in combination with insensitivity to exogenous reference points ($\eta_{i,N}^j = 0$ for all $j$) and a gain-loss utility function $\mu(\cdot)$ satisfying A0-2, A3’, and A4. For every social comparison between $i$ and another person $j$, assuming $\hat{\eta}_{i,A}^i \geq 0$, we have

\[
\begin{align*}
\alpha_{ij} &= \lambda_i \hat{\eta}_{i,A}^i - \hat{\eta}_{j,A}^j, \\
\beta_{ij} &= \lambda_i \hat{\eta}_{i,A}^i - \hat{\eta}_{i,A}^i.
\end{align*}
\]

Within the same context and assuming $\hat{\eta}_{i,A}^i > 0$, $\alpha_{ij}$ and $\beta_{ij}$ are positively correlated:

\[
\begin{align*}
\alpha_{ij} &= \lambda_i \hat{\eta}_{i,A}^i - \hat{\eta}_{j,A}^j, \\
\beta_{ij} &= \lambda_i \hat{\eta}_{i,A}^i - \hat{\eta}_{i,A}^i.
\end{align*}
\]

**Proof.** See appendix. \hfill \square

First note that in Theorem 5.1 $\alpha_{ij}$ and $\beta_{ij}$ depend on the group (represented by person $j$) to which DM compares herself in addition to the context in which the decision takes place. However, $\eta_{i,A}^j(n) = \eta_{k,A}^j(n)$, for all $j \neq k$ except $j = i$, implies $\alpha_{ij} = \alpha_i$ and $\beta_{ij} = \beta_i$ for all $j \neq i$. In the following we permit context-dependent preferences, though, which is why we continue to write $\alpha_{ij}$ and $\beta_{ij}$.

**Corollary 5.1.** For any $\lambda_i \geq 1$ and given a particular context, $\hat{\eta}_{i,A}^i \geq \hat{\eta}_{j,A}^j$ for any $j \neq i$, if and only if $\alpha_{ij} \geq \beta_{ij}$ for the same $j$ in this context.

**Corollary 5.2.** $u^{FS}_i(x)$ satisfies monotonicity assumption M if and only if

\[
\beta_i \equiv \frac{1}{n-1} \sum_{j \neq i} \beta_{ij} < 1.
\]

The reverse implication follows by solving (5.5) for $\beta_{ij}$ and then proving the contraposition $\hat{\eta}_{i,A}^i < \hat{\eta}_{i,A}^j \Rightarrow \alpha_{ij} < \beta_{ij}$.\footnote{The reverse implication follows by solving (5.5) for $\beta_{ij}$ and then proving the contraposition $\hat{\eta}_{i,A}^i < \hat{\eta}_{i,A}^j \Rightarrow \alpha_{ij} < \beta_{ij}$.}
Similar to Proposition 4.2, Corollary 5.2 gives an upper bound for $\beta_i \equiv \frac{1}{n-1} \sum_{j \neq i} \beta_{ij}$. In heterogeneous groups DM may accordingly excessively favor individual persons, so that $\beta_{ij} \geq 1$ for some $j$, and still preserve the monotonicity of her preferences.

Our model generalizes Fehr-Schmidt preferences because $u_i^{ENA}(x|r)$ is context-dependent. This is consistent with the empirical observations by Loewenstein et al. (1989): A utility function similar to the aggregate shown in Figure 1 applies to social comparisons among persons with a positive relationship (e.g., friends) or a small social distance (e.g., neighbors or family). However, for business decisions Loewenstein et al. (1989) report utility functions more in line with prospect theory. Given their results’ apparent contradiction to prospect theory in non-business contexts, Loewenstein et al. (1989) leave an explanatory gap for the Fehr-Schmidt model to fill ten years later.

Yet the Fehr-Schmidt model is incomplete in that it does not account for other persons’ needs, making its predictions susceptible to refutation under a framing that stresses such exogenous reference points. The findings by Engelmann and Strobela (2004) exploit this weakness and thus reveal decisions that appear to be more in line with maximin preferences. Furthermore, the ENA-model predicts a correlation between the Fehr-Schmidt parameters that may not exist in practice (Rutledge et al., 2016). However, stressing empathy with others’ needs in addition to (or instead of) others’ ambitions can overcome these problems, as we will demonstrate in the following sections.

6 Social value orientation and reference dependence

DM’s social value orientation (e.g., Chen and Fischbacher, 2016; Liebrand and McClintock, 1988; Murphy et al., 2011) reveals her preferences for distributive fairness with respect to payoffs for herself and other persons. Specifically, the social value orientation indicates DM’s preferred allocation among a set of outcomes $X^{SVO} = \{x| \sum_j x_j^2 = \rho^2\}$. In other words, the available outcomes $x$ are arranged on a sphere in $\mathbb{R}^n$ with radius $\rho$, so that $\sum_j x_j^2 = \rho^2$. For this purpose, DM’s utility function $u_i^{SVO}(x)$ is defined as follows:

$$u_i^{SVO}(x) = a x_i + \sum_{j \neq i} b_j x_j$$

(6.1)

where $a$ and $b_j$ are real numbers. Let $x^*$ maximize $u_i^{SVO}(x)$ subject to $\sum_j x_j^2 = \rho^2$ (see Lemma 6.1 below). Then DM’s social value orientation towards person $j$, given by

$$\angle_{i,SVO}^j = \arctan(\frac{b_j}{a})$$

(6.2)

$\text{29}$M also implies a lower bound for $\alpha_i \equiv \frac{1}{n-1} \sum_{j \neq i} \alpha_{ij}$ of $\alpha_i > -1$. However, $\hat{\eta}_{i,A} \geq \hat{\eta}_{j,A}$ for all $j$ guarantees $\alpha_i > 0$. $\text{30}$Fehr and Schmidt (1999, p.821) explicitly mention Loewenstein et al. (1989) as “strong evidence for the importance of relative payoffs” and use their results to justify the assumptions for $\alpha_i$ and $\beta_i$ (Fehr and Schmidt, 1999 pp. 823f.). Fehr and Schmidt (1999) also briefly mention loss aversion, but do not elaborate on similarities (or differences) of their model to prospect theory. $\text{31}$For an alternative interpretation see Fehr et al. (2006).
can be determined by locating \((x_i^*, x_j^*)\) on the two-dimensional subset of \(X^{SVO}\) on which only \(x_i\) and \(x_j\) can attain non-zero values.\(^{32}\) If \(a = 0\), then \(\angle^j_{i,SVO}\) is equal to 90°.

**Lemma 6.1.** On \(X^{SVO}\), \(u^{SVO}_i(x)\) is maximized by \(x^*\) with

\[
x_i^* = \frac{a\rho}{\sqrt{a^2 + \sum_{k \neq i} b_k^2}} \tag{6.3}
\]

\[\forall j : x_j^* = \frac{b_j\rho}{\sqrt{a^2 + \sum_{k \neq i} b_k^2}} \tag{6.4}\]

*Proof.* Trivial. \(\Box\)

\(^{32}\)The indifference curves of \(u^{SVO}_i(x)\) are lines in the two-dimensional space spanned by \(x_i\) and \(x_j\). At the maximum value, the indifference curve is tangential to the circle of outcomes given by \(x_i^2 + x_j^2 = \rho^2\).

Liebrand and McClintock (1988) and other authors associate personality types (such as altruistic or competitive) with specific angles \(\angle^j_{i,SVO}\). In particular, Liebrand and McClintock (1988) identify each personality type with one of eight cardinal directions on the circle of outcomes represented by a specific allocation, similar to a moral compass. However, in the practice of experimental investigations, only angles between \(-45°\) (competitive) and 90° (altruistic) are used. This is so probably because of monotonicity assumptions:

**Lemma 6.2.** If \(u^{SVO}_i(x)\) satisfies monotonicity assumptions \(M\) and \(E\), then \(\angle^j_{i,SVO} \in [-45°, 90°]\) for all \(j\).

*Proof.* Trivial. \(\Box\)

With this theoretical basis, we can now examine social value orientations from the perspective of the ENA-model. We discuss two model variants: one based on empathy with ambitions (EA), the other based on empathy with needs (EN).

### 6.1 Social value orientation and empathy with ambitions

We first assume again that \(\eta^j_{k,N} = 0\) for all \(j\) and show that \(u^{EA}_i(x|r)\) belongs to the class of utility functions given by \(u^{SVO}_i(x)\).\(^{33}\)

**Proposition 6.1.** If \(\eta^k_{i,N} = 0\) for all \(k\), then

\[
u^{EA}_i(x|r) = x_i + \eta^j_{i,A}(n) \sum_{k \neq i} \mu_i(x_i|x_k) + \sum_{k \neq i} \eta^k_{i,A}(n) \mu_i(x_k|x_i)
\]

yields DM’s social value orientation, which with respect to each \(j\) is given by the angle \(\angle^j_{i,A}(\lambda_i, \eta^j_{i,A}, \eta^j_{i,A}) \in [-45°, 90°]\). We have

\[^{33}\text{This is to say that social value orientations are more general than reference-dependent preferences in this specific application, just as reference-dependent preferences are more general than Fehr-Schmidt preferences.}\]
\[ \angle_{i,A}^{j}(\lambda, \eta_{i,A}^{j}(n), \eta_{j,A}^{j}(n)) = \arctan(b_j/a) \] (6.5)

with

\[ b_j = \begin{cases} 
  b_j^+ & \text{if } x_i \geq x_j \\
  b_j^- & \text{if } x_i < x_j 
\end{cases} \] (6.6)

and

\[ a = 1 - \sum_{k \neq i} b_k. \] (6.7)

**Proof.** The identification of \( a \) and \( b_j \) is straightforward, but requires distinguishing between several cases depending on the relative position of \( x_i \) and \( x_j \) compared to each other and (for \( x_i \)) to the other persons’ outcomes.

\( u^{EA}_i(x|r) \) is compatible with the utility function used by [Charness and Rabin (2002)](https://doi.org/10.1093/restud/79.3.443), because the sum of the weights \( a \) and \( b_k, \ k \neq i \), is equal to 1.\(^{34}\) This is remarkable considering that the sensitivity parameters \( \eta^{k}_{i,A}(n) \) can vary freely. \( u^{EA}_i(x|r) \) has this property because all its utility components, except for the consumption utility, depend on the difference between \( x_i \) and another payoff.

The following proposition establishes how \( \angle_{i,A}^{j}(\lambda, \eta_{i,A}^{j}(n), \eta_{j,A}^{j}(n)) \) is affected by changes to \( \eta_{i,A}^{j}(n) \), \( \eta_{i,A}^{j}(n) \), and DM’s sensitivity to a third person \( \eta^{k}_{i,A}(n) \):

**Proposition 6.2.** Given \( u^{EA}_i(x|r) \), DM’s social value orientation with respect to person \( j \) satisfies the following

\[ \frac{\partial}{\partial \eta_{i,A}^{j}(n)} \angle_{i,A}^{j} < 0 \] (6.8)

\[ \frac{\partial}{\partial \eta_{j,A}^{j}(n)} \angle_{i,A}^{j} > 0 \] (6.9)

\[ \frac{\partial}{\partial \eta^{k}_{i,A}(n)} \angle_{i,A}^{j} > 0 \] (6.10)

**Proof.** \( \arctan(x) \) is an increasing function on \( ]-45^\circ, 90^\circ[ \). The proposition therefore follows immediately by examining the effects of \( \eta_{i,A}^{j}(n), \eta_{j,A}^{j}(n), \) and \( \eta^{k}_{i,A}(n) \) on \( b_j \) and \( a \) : \( \eta_{i,A}^{j}(n) \) decreases \( b_j \), and thus increases \( a \), for a combined decrease of \( \angle_{i,A}^{j} \). \( \eta_{i,A}^{j}(n) \) increases \( b_j \), and thus decreases \( a \), for a combined increase of \( \angle_{i,A}^{j} \). \( \eta^{k}_{i,A}(n) \) only decreases \( a \), but this still increases \( \angle_{i,A}^{j} \), though more weakly than \( \eta_{i,A}^{j}(n) \).

\(^{34}\)In the notation of [Charness and Rabin (2002)](https://doi.org/10.1093/restud/79.3.443) we have \( b_j = \rho r + \sigma s + \theta q \), where \( r,s,q \) are indicator variables. \( \rho \) and \( \sigma \) capture DM’s envy and guilt, respectively, similar to the Fehr-Schmidt parameters. \( \theta \) measures DM’s propensity for reciprocal action depending on \( j \)’s past behavior.
The first two results are intuitive: The more sensitive DM is to her own (the other person’s) ambitions, the lower (higher) the outcome share she allocates to the other person. The third result is an indirect effect: DM’s higher sensitivity to a third person k’s ambitions affects her decision even when allocating payoffs only between herself and person j. The increase in $\eta_{i,A}(n)$ means that DM has a stronger interest in reducing the payoff difference $x_i - x_k$. But the only way to do so is if DM decreases $x_i$, which also makes her worse off relative to $x_j$ and thus increases $\angle_{i,A}$.

The effect of loss aversion on social value orientations based on ambitions is less clear. A high value of $\lambda_i$ induces DM to avoid extreme allocations that assign to her an outcome share that is higher (or lower) than that of all other persons. However, this pull towards equal shares (effectively inequality aversion) does not necessarily persist if $x_i$ is larger than some persons’ outcomes, but smaller than those of others. It does persist, however, if reasonable additional assumptions are fulfilled, such as a higher sensitivity for own ambitions than for others’ ambitions combined with a roughly equal sensitivity for the ambitions of other persons:

**Proposition 6.3.** Given $u_i^{EA}(x|r)$, if $\eta_{j,A}(n) \geq \eta_{i,A}(n)$ for all $j$ and $\eta_{i,A}(n) \approx \eta_{k,A}(n)$ for all $j,k \neq i$, then for all $j$:

$$x_i \lesssim x_j \Leftrightarrow \frac{\partial}{\partial \lambda_i} \angle_{i,A} \lesssim 0 \quad (6.11)$$

**Proof.** See appendix.

We finish this section by examining the limits of social value orientations based on ambition for extreme combination of the sensitivity parameters. If $\eta_{i,A}(n) = 0$ for all $j \neq i$, then $b_j$ is negative and $\angle_{i,A} \in [45^\circ, 0^\circ]$, implying competitiveness. If $\eta_{i,A}(n) = 0$, then $b_j$ is positive and $\angle_{i,A} \in [0^\circ, 90^\circ]$, including individualistic, prosocial, and altruistic types. By dropping monotonicity assumption M, so that DM (irrationally) places much more weight on others’ ambitions than her own, we can extend this range to $\angle_{i,A} \in [0^\circ, 135^\circ]$. This adds “self-sacrificing” or “martyr” personalities to the accessible types, who aim to maximize $x_j - x_i$.

### 6.2 Social value orientation and empathy with needs

Contrary to the previous section we assume now that $\eta_{i,A}(n) = 0$, but $\eta_{i,N}(n) \geq 0$ for all $j$ (including $i$). DM accordingly is insensitive to ambitions, but possibly empathizes with others’ needs. We obtain

$$u_i^{EN}(x|r) = x_i + \sum_j \eta_{i,N}(n)\mu_i(x_j|r_j) \quad (6.12)$$

If $\mu_i(\cdot)$ satisfies A3’, then this is equal to

$$u_i^{EN}(x|r) = x_i + \sum_{j: x_j \geq r_j} \eta_{i,N}(n)(x_j - r_j) - \lambda_i \sum_{j: x_j < r_j} \eta_{i,N}(n)(r_j - x_j) \quad (6.13)$$
Obviously, \( u^E_i(x|r) \) belongs to the class of utility functions \( u^{SVO}_i(x) \) with

\[
a = 1 + \mathbb{1}_{x_i \geq r_i} \eta_{i,N}^\ell(n) + \mathbb{1}_{x_i < r_i} \lambda_i \eta_{i,N}^\ell(n) > 0 \quad (6.14)
\]

\[
\forall j \neq i : b_j = \mathbb{1}_{x_j \geq r_j} \eta_{i,N}^j(n) + \mathbb{1}_{x_j < r_j} \lambda_i \eta_{i,N}^j(n) \geq 0 \quad (6.15)
\]

and

\[
\angle_{i,N}^j(\lambda_i, \eta_{i,N}^\ell(n), \eta_{i,N}^j(n)) = \arctan\left(\frac{b_j}{a}\right) \quad (6.16)
\]

\[
= \arctan\left(\frac{\mathbb{1}_{x_j \geq r_j} \eta_{i,N}^\ell(n) + \mathbb{1}_{x_j < r_j} \lambda_i \eta_{i,N}^j(n)}{1 + \mathbb{1}_{x_i \geq r_i} \eta_{i,N}^\ell(n) + \mathbb{1}_{x_i < r_i} \lambda_i \eta_{i,N}^\ell(n)}\right) \quad (6.17)
\]

It is easy to see that \( u^E_i(x|r) \) accommodates social value orientations within the range \([0^\circ, 90^\circ]\), representing individualistic, prosocial, and altruistic DMs. Note, however, that \( a \) and \( b_j \) are now independent of each other. This means that \( u^E_i(x|r) \) is incompatible with the model used by Charness and Rabin (2002).

In analogy to Proposition 6.2 the following holds:

**Proposition 6.4.** Given \( u^E_i(x|r) \), DM’s social value orientation with respect to person \( j \) satisfies the following

\[
\frac{\partial}{\partial \eta_{i,N}^\ell(n)} \angle_{i,N}^j < 0 \quad (6.18)
\]

\[
\frac{\partial}{\partial \eta_{i,N}^j(n)} \angle_{i,N}^j > 0 \quad (6.19)
\]

\[
\frac{\partial}{\partial \eta_{i,N}^k(n)} \angle_{i,N}^j = 0 \quad (6.20)
\]

**Proof.** Trivial. \( \square \)

\( \angle_{i,N}^j \) is independent of DM’s sensitivity regarding a third person \( k \)’s needs, because the payoff difference \( x_k - r_k \) cannot be affected by changing \( x_i \) or \( x_j \).

Like with empathy with ambitions, the effect of \( \lambda_i \) on \( \angle_{i,N}^j \) depends on the relative position of \( x_i \) and \( x_j \), this time compared to the reference outcomes \( r_i \) and \( r_j \), respectively:

**Proposition 6.5.** Given \( u^E_i(x|r) \), DM’s social value orientation with respect to person \( j \) satisfies the following

\[
x_i \geq r_i \land x_j \geq r_j \iff \frac{\partial}{\partial \lambda_i} \angle_{i,N}^j = 0 \quad (6.21)
\]

\[
x_i < r_i \land x_j \geq r_j \iff \frac{\partial}{\partial \lambda_i} \angle_{i,N}^j < 0 \quad (6.22)
\]

\[
x_j < r_j \iff \frac{\partial}{\partial \lambda_i} \angle_{i,N}^j > 0 \quad (6.23)
\]

\( ^{35} \)Remember that the utility level can be set freely, permitting us to drop any terms not including \( x_i \) or \( x_j \).
Table 1: Differences between social value orientations 1) based on ambitions or 2) based on needs.

| Range of $\angle_{i,R}$ | Ambitions ($u_{i}^{EA}(x|r)$) | Needs ($u_{i}^{EN}(x|r)$) |
|--------------------------|-------------------------------|-----------------------------|
| w. (M)                   | $]-45^\circ, 90^\circ[$       | $][0^\circ, 90^\circ[$      |
| w/o. (M)                 | $]-45^\circ, 135^\circ[$     | $][0^\circ, 90^\circ[$      |
| $\eta_{j,R} = 0$         | $][0^\circ, 90^\circ[$       | $\{0^\circ\}$              |

<table>
<thead>
<tr>
<th>Relevant payoff comparison</th>
<th>$x_j$ vs. $x_i$</th>
<th>$x_j$ vs. $r_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial}{\partial \eta_{i,R}(\alpha)} \angle_{i,R}$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$\frac{\partial}{\partial x_{i}} \angle_{i,R}$</td>
<td>$&lt; 0$ if $x_{i} &lt; x_{j}$</td>
<td>$x_{i} &lt; r_{i} \land x_{j} \geq r_{j}$</td>
</tr>
<tr>
<td></td>
<td>$= 0$ if $x_{i} = x_{j}$</td>
<td>$x_{i} \geq r_{i} \land x_{j} \geq r_{j}$</td>
</tr>
<tr>
<td></td>
<td>$&gt; 0$ if $x_{i} &gt; x_{j}$</td>
<td>$x_{j} &lt; r_{j}$</td>
</tr>
</tbody>
</table>

$\alpha$ and $b_{j}$ independent No ($a = 1 - \sum_{k\neq i} b_{k}$) Yes

Proof. Trivial, noting again that $\arctan(x)$ is a monotonic function.

Lifting monotonocity assumption M has no effect on $u_{i}^{EN}(x|r)$ because this property is already implied by $\eta_{j,N}(n) \geq 0$ for all $j$. As a result, a competitive (negative) angle $\angle_{j,N}$ is impossible, even if $\eta_{j,N}(n) = 0$ (in which case $\angle_{j,N} = 0^\circ$).

6.3 Needs or ambitions or both?

By presenting two competing models of social value orientations, we have demonstrated that the associated decision tasks only insufficiently determine the motives for choosing particular outcomes. As soon as observed social value orientation angles are employed to predict decisions in other circumstances, inconsistencies will occur as a person motivated primarily by needs suddenly acts differently than a person motivated primarily by ambitions.

But $u_{i}^{EA}(x|r)$ and $u_{i}^{EN}(x|r)$ do not make completely identical predictions and can therefore be tested against each other. Table 1 lists differences between the two models. For one thing, $u_{i}^{EA}(x|r)$ permits a broader range of angles than $u_{i}^{EN}(x|r)$, whether monotonocity assumption M is satisfied or not. Although most observed social value orientations fall within the common range of both models (between $0^\circ$ and $90^\circ$), the occasional outlier with a negative angle supports empathy with ambitions. Such “competitive” subjects are incompatible with $u_{i}^{EN}(x|r)$, with the consequence that subjects who do not empathize with others’ needs ($\eta_{i,N}(n) = 0$) are always associated with an angle of $0^\circ$. This difference between the models is also reflected in the relevant payoff comparison between the other person’s outcome $x_{j}$ and the reference point. If DM empathizes with $j$’s ambitions, she compares $x_{j}$ to her own outcome $x_{i}$. If she empathizes with $j$’s needs, she compares $x_{j}$ to $j$’s exogenous reference point $r_{j}$. Testing whether DM accesses information only on $x_{j}$ or also on $r_{j}$ can further help distinguish empathy with needs from empathy with ambitions.

More subtle differences between $u_{i}^{EA}(x|r)$ and $u_{i}^{EN}(x|r)$ concern the partial derivatives of $\angle_{i,R}(\cdot)$ with respect to the sensitivity to a third person $k$’s needs or ambitions $\eta_{k,R}(n)$ or...
DM’s extent of loss aversion $\lambda_i$. For $u^{EN}_i(x|r)$, $\angle_{i,R}^j(\cdot)$ is independent of $\eta_{i,N}^k(n)$. Accordingly, if the presence of a third person affects DM’s social value orientation angle, then she likely empathizes with others’ ambitions. With respect to loss aversion $u^{EA}_i(x|r)$ predicts that empathy with ambitions combined with high loss aversion yields “prosocial” outcomes close to 45°. The corresponding prediction of $u^{EN}_i(x|r)$ depends on the exogenous reference points $r_i$ and $r_j$, which can be experimentally manipulated. For example, if both reference points are small compared to the range of possible outcomes, $\lambda_i$ should be uncorrelated with DM’s social value orientation, if it is based on empathy with needs.

Finally, assume that the experimenter ignores both empathy models and instead fits the data to $u^{SVO}_i(x)$ to empirically estimate $a$ and $b_j$. If both parameters are independent of each other, DM cannot have based her decision on social comparisons. This may also be considered a conceptual flaw of empathy with ambitions, because $u^{EA}_i(x|r)$ is falsely assumed to be additively separable.

If the goal is to develop a model that applies in different contexts, a better alternative to choosing one empathy model over the other is to combine both in the ENA model presented in Section 4 and use this model to calculate DM’s social value orientation angle. For simplicity assume that $n = 2$ as well as $x_i \geq r_i$ and $x_j \geq r_j$ for all $x \in X$.

Then
\begin{equation}
\angle_{i,NA}^j = \arctan\left(\frac{b_j}{a}\right)
\end{equation}

with
\begin{equation}
b_j = \begin{cases} 
\eta_{i,N}^j - \eta_{i,A}^j + \lambda_i \eta_{i,A}^j & \text{if } x_i \geq x_j \\
\eta_{i,N}^j - \lambda_i \eta_{i,A}^j + \eta_{i,A}^j & \text{if } x_i < x_j
\end{cases}
\end{equation}

and
\begin{equation}
a = 1 + \eta_{i,N}^j + \eta_{i,N}^j - b_j.
\end{equation}

7 Empathy in public good provision

We now demonstrate in the example of a linear public goods game, how empathy with others’ needs affects contribution behavior. In particular, we show that the ENA model can accommodate the frequently observed “hump-shaped” preferences with respect to contributions conditional on the group average [Fischbacher et al., 2001]. In this game, each player $i$ in a group of $n$ players is given an endowment $e_i$ in order to make a voluntary individual contribution of $g_i$ to a public account. This contribution is multiplied by a factor $m \in ]1/n, 1[$ and then paid to all players in the group (including $i$). Player $i$’s payoff is accordingly given by

---

36 This holds not only for empathy with ambitions, but also for inequality aversion and the model used by Charness and Rabin (2002). Even if $\eta_{i,A}^j = 0$, $a$ and $b_j$ are (negatively) correlated.

37 Extensions to larger groups and the other cases with respect to the exogenous reference points are straightforward.

38 Different ways of framing this game are used in the literature. See e.g. Cartwright (2016).
\[
x_i(g_i, g-1) = e_i - g_i + m \sum_{j=1}^{n} g_j
\]  
\hspace{1em} (7.1)

A common method to elicit a subject’s strategy for this game (e.g. Fischbacher et al., 2001) is to ask for a contribution \( g_i \) conditional on the average contribution of the other group members, \( \bar{g} \equiv \frac{1}{n-1} \sum_{j \neq i} g_j \). This gives \( g_i \) as a function of \( \bar{g} \). We can apply the ENA model to calculate \( g_i(\bar{g}) \), representing player \( i \)’s utility for each game outcome. All other group members \( j \) are treated as identical with respect to their endowment \( e_j \), their exogenous reference point \( r_j \), as well as their resulting contribution \( g_j = \bar{g} \) and outcome \( x_j \). This implies:

\[
x_i(g_i, \bar{g}) = e_i - (1 - m) g_i + (n - 1) m \bar{g} \tag{7.2}
\]

\[
x_j(g_i, \bar{g}) = e_j + m g_i + ((n - 1) m - 1) \bar{g} \tag{7.3}
\]

With respect to a linear public goods game, \( u^{ENA}_i(x|r) = u^{ENA}_i(g_i, \bar{g}|r) \) therefore yields the following:

\[
u^{ENA}_i(g_i, \bar{g}|r) = x_i(g_i, \bar{g}) + \eta^i_{N} \mu_i(x_j(g_i, \bar{g})|r_i) + (n - 1) \eta^j_{N} \mu_i(x_j(g_i, \bar{g})|r_j) + \eta^i_{A} (n - 1) \mu_i(x_j(g_i, \bar{g})|x_j(g_i, \bar{g})) + (n - 1) \eta^j_{A} \mu_i(x_j(g_i, \bar{g})|x_i(g_i, \bar{g})) \tag{7.4}
\]

Player \( i \)’s reference points divide the strategy space \( G = \{(g_i, \bar{g})|g_i \geq 0 \land \bar{g} \geq 0\} \) into different areas, depending on whether or not the associated outcome \( x \) satisfies each reference point. In particular the following dividing lines are of interest:

\[
x_i(g_i, \bar{g}) = r_i \iff \gamma^1_N(\bar{g}) = \frac{e_i - r_i}{1 - m} + \frac{(n - 1) m}{1 - m} \bar{g} \tag{7.5}
\]

\[
x_j(g_i, \bar{g}) = r_j \iff \gamma^2_N(\bar{g}) = \frac{r_j - e_j}{m} - (n - 1 - 1/m) \bar{g} \tag{7.6}
\]

\[
x_i(g_i, \bar{g}) = x_j(g_i, \bar{g}) \iff \gamma_A(\bar{g}) = e_i - e_j + \bar{g} \tag{7.7}
\]

The first two dividing lines arise from exogenous reference points associated with needs (N), the third line represents the social comparison relevant for ambitions (A). To simplify our argument, we therefore again analyze the effect of ambitions separately from that of needs.

\[\text{This only gives the player’s preferences. Her actual contribution may still depend on strategic considerations that are beyond the scope of the decision-theoretic approach used in this paper.}\]
7.1 Empathy with ambitions

Given $\eta_{i,N}^j = 0$ for all $j$ and assuming a risk-neutral DM, A3' implies:

$$u_{i}^{EA}(g_i, \bar{g}|r) = e_i - (1 - m)g_i + (n - 1)m\bar{g}$$

$$+ (n - 1) \begin{cases} 
(\eta_{i,A}^i - \lambda_i\eta_{i,A}^i)(\gamma_A(\bar{g}) - g_i) & \text{if } g_i \leq \gamma_A(\bar{g}) \\
(\lambda_i\eta_{i,A}^i - \eta_{j,A}^i)(\gamma_A(\bar{g}) - g_i) & \text{if } g_i > \gamma_A(\bar{g}) \end{cases} \tag{7.8}$$

$g_i(\bar{g})$ can attain three possible cases depending on whether $u_{i}^{EA}(g_i, \bar{g}|r)$ is increasing or decreasing in $g_i$ above and below the dividing line $\gamma_A(\bar{g})$.

**Proposition 7.1.** Given a risk-neutral DM who is sensitive to ambitions, but not needs, the following holds:

1. $\eta_{i,A}^j < \eta_{i,A}^i \lambda_i + \frac{1-m}{\lambda_i(n-1)} \iff g_i(\bar{g}) = 0$
2. $\eta_{i,A}^j + \frac{1-m}{\lambda_i(n-1)} < \eta_{i,A}^j < \lambda_i\eta_{i,A}^i + \frac{1-m}{n-1} \iff g_i(\bar{g}) = \gamma_A(\bar{g})$
3. $\eta_{i,A}^j > \lambda_i\eta_{i,A}^i + \frac{1-m}{n-1} \iff g_i(\bar{g}) = \infty$

**Proof.** Calculate $\frac{\partial}{\partial g_i} u_{i}^{EA}(g_i, \bar{g}|r)$ (assuming A3') for all cases to verify the result. \hfill \Box

In words, DM is either selfish (1.), a conditional cooperator (2.), or altruistic (3.), depending on her sensitivity $\eta_{i,A}^j$ to others’ ambitions. As most participants in the experiment by [Fischbacher et al. (2001)] are either selfish or (more or less) conditional cooperators, one might be satisfied with modeling contribution behavior by means of social comparisons. However, $u_{i}^{EA}(g_i, \bar{g}|r)$ cannot accommodate the non-negligible share of “hump-shaped” preferences, which specify that the subject only makes a positive contribution given a medium range of $\bar{g}$.

7.2 Empathy with needs

Empathy with others’ needs is probably most in line with the “warm glow” of giving postulated by [Andreoni (1990)] in public good provision. Contrary to the previous section, we now assume $\eta_{i,A}^j = 0$, but $\eta_{i,N}^j \geq 0$ for all $j$. With two dividing lines, $\gamma_{N1}(\bar{g})$ and $\gamma_{N2}(\bar{g})$, this still leaves four different cases to consider, depending on whether $x_i$ and $x_j$ are above or below their respective exogenous reference points $r_i$ and $r_j$.

---

40If any of the conditions given below holds with equality, then DM’s best response consists in a range of values. For example, if $\eta_{i,A}^j = \frac{\eta_{i,A}^i}{\lambda_i} + \frac{1-m}{\lambda_i(n-1)}$, then DM maximizes her utility with any $g_i \in [0, \gamma_A(\bar{g})]$. 

21
Proposition 7.2. Given a risk-neutral DM who is sensitive to needs, but not ambitions, the following holds:

1. \( \eta^i_{i,N} < (1 + \eta^i_{i,N}) \frac{1-m}{\lambda_i(n-1)m} \Leftrightarrow g_i(\bar{g}) = 0 \)

2. \( (1 + \eta^i_{i,N}) \frac{1-m}{\lambda_i(n-1)m} < \eta^j_{i,N} < (1/\lambda_i + \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = \min\{\gamma_{N1}(\bar{g}), \gamma_{N2}(\bar{g})\} \)

3. \( (1/\lambda_i + \eta^i_{i,N}) \frac{1-m}{(n-1)m} < \eta^j_{i,N} < (1 + \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = \gamma_{N1}(\bar{g}) \)

4. \( (1 + \eta^i_{i,N}) \frac{1-m}{\lambda_i(n-1)m} < \eta^j_{i,N} < (1 + \lambda_i \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = \max\{\gamma_{N1}(\bar{g}), \gamma_{N2}(\bar{g})\} \)

5. \( \eta^j_{i,N} > (1 + \lambda_i \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = \infty \)

Proof. Calculate \( \frac{\partial}{\partial g_i} u_i^{E,N}(g_i, \bar{g}|r) \) (assuming A3') for all cases to verify the result. \( \square \)

Empathy with needs, but not ambitions, results in five preference types, depending on DM’s sensitivity parameter \( \eta^i_{i,N} \). In addition to selfish (1.), conditionally cooperating (3.), and altruistic (5.) types, DM’s preferences may now also be “hump-shaped” (2.) or “v-shaped” (4.). Depending on how \( \gamma_{N1}(\bar{g}) \) and \( \gamma_{N2}(\bar{g}) \) intersect the strategy space \( G \), even more shapes may emerge. For example, DM can be selfish for low group contributions, but conditionally cooperate once the group contribution exceeds a specific minimum. Moreover, if \( r_j = e_j \), then DM’s preferences mimic those for empathy with ambitions.\(^{42}\)

Corollary 7.1. Let \( r_j = e_j \). Then, given a risk-neutral DM who is sensitive to needs, but not ambitions, the following holds:

1. \( \eta^j_{i,N} < (1/\lambda_i + \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = 0 \)

2. \( (1/\lambda_i + \eta^i_{i,N}) \frac{1-m}{(n-1)m} < \eta^j_{i,N} < (1 + \lambda_i \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = \gamma_{N1}(\bar{g}) \)

3. \( \eta^j_{i,N} > (1 + \lambda_i \eta^i_{i,N}) \frac{1-m}{(n-1)m} \Leftrightarrow g_i(\bar{g}) = \infty \)

This not only suggests that a subject with hump-shaped preferences empathizes with others’ needs, but also that the reference point she assumes for the other group members (and possibly for herself) is higher than \( e_j \). This is especially remarkable considering that \( e_j \) is the payoff in the game’s unique Nash equilibrium (\( g_j = 0 \) for all \( j \)). These subjects accordingly expect the other group members to cooperate to at least some extent and therefore adjust their strategy accordingly. The corollary might also make it possible to use the strategy method to reveal framing effects (e.g., \( \text{Cartwright, 2016} \)) by comparing the frequency of subjects displaying hump-shaped preferences.

\(^{41}\)Again, if any of the conditions holds with equality, then DM can choose \( g_i \) from a range of values.

\(^{42}\)To obtain the corollary, note that assuming \( r_j = e_j \) causes \( \gamma_{N2}(\bar{g}) \) to intersect with the origin. In Proposition 7.2, this collapses hump-shaped preferences to selfish contributions and v-shaped preferences to conditional cooperation.
Figure 2: Best response functions by preference type for \textit{Fischbacher et al. (2001)} based on $u_i^{EA}(g, \bar{g}|r)$ (ambitions), $u_i^{EN}(g, \bar{g}|r)$ (needs) and a combined model with sensitivity to own ambitions, but others’ needs.
7.3 Own ambitions, others’ needs

In addition to the occasional hump-shaped preference, Fischbacher et al. (2001) also observe that the increasing part of a subject’s best-response function often has an incline of 1, i.e. if the average contribution increases by 1, so does the player’s contribution. Yet the incline of \( \gamma_{N1}(\bar{g}) \) is always strictly greater than 1 and possibly much steeper, whereas \( \gamma_A(\bar{g}) \) matches the observed incline exactly. This means that, for conditional cooperators, sensitivity to one’s own ambitions provides a better fit with the data than sensitivity to one’s own needs. Hump-shaped preferences whose increasing part has an incline of 1 therefore suggest that both needs and ambitions influence the decision.

Figure 2 illustrates this point graphically. The figure displays the best-response functions by preference type for the game used by Fischbacher et al. (2001) in three scenarios. The first scenario (top row) assumes \( u^{EA}(g_i, \bar{g}|r) \) (ambitions only), the second (middle row) \( u^{EN}(g_i, \bar{g}|r) \) (needs only), the third (bottom row) combines aspects from both models by assuming sensitivity to own ambitions \( (\eta_{i,A}^j > 0) \), but others’ needs \( (\eta_{i,N}^j > 0) \). In the latter two cases, DM’s reference point is set equal to her endowment \( (r_i = e_i = 20) \), so that \( \gamma_{N1}(\bar{g}) \) starts at the origin. The other players’ reference point is set to \( r_j = 26 \), so that \( \gamma_{N2}(\bar{g}) \) passes through the point \( (\bar{g}, g_i) = (10, 10) \) and therefore roughly divides the strategy space in half.\(^43\)

Note that the parameters \( \eta_{i,N}^j, \eta_{i,N}^j, \) and \( \lambda_i \) do not affect the shape of the best-response function; they only determine the player’s preference type. While a strong theoretical result, this limits the model’s explanatory power in the light of the high variability of shapes observed by Fischbacher et al. (2001).

Hump-shaped preferences in the form observed by Fischbacher et al. (2001) speak for a combined effect of own ambitions and empathy with others’ needs on the decision. Yet this player type is associated with only a relatively low value for the sensitivity parameter \( \eta_{i,N}^j \), just shy of selfishly contributing nothing. Such players do not necessarily aim to satisfy others’ needs and likely do not go out of the way to do so. And while they appear to realize that cooperation benefits themselves as well as the group, they also prefer to benefit more than others if possible.

8 Conclusion

We have presented a theoretical model of other-regarding preferences based on reference dependence. In the model, the decision-maker is not only sensitive to personal gains and losses, but also her social status expressed by her relative payoff compared to other people. Moreover, she empathizes with other persons’ gains and losses, i.e. their needs, as well as their ambitions to increase their social status. Each of the four main components of this ENA model is justified by experimental observations, as summarized in Table 2. Prospect theory provides ample support of the claim that a decision-maker is sensitive to her own needs. Hump-shaped preferences in linear public goods games can only be explained by the ENA model, if the decision-maker is sensitive to others’ needs. Sensitivity to own ambitions becomes apparent in competitive social value orientations. Finally, sensitivity to others’

\(^{43}\) Choose \( r_j = 24 \) to achieve a “hump” starts and ends with a contribution of zero.
Table 2: Experimental observations and associated component of the ENA model

<table>
<thead>
<tr>
<th>Observation</th>
<th>Model component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss aversion</td>
<td>Sensitivity to own needs ($\eta_{i,N}^i &gt; 0$)</td>
</tr>
<tr>
<td>Hump-shaped preferences in linear PG</td>
<td>Sensitivity to others’ needs ($\eta_{i,N}^j &gt; 0$)</td>
</tr>
<tr>
<td>Competitive social value orientation</td>
<td>Sensitivity to own ambitions ($\eta_{i,A}^i &gt; 0$)</td>
</tr>
<tr>
<td>Inequality aversion</td>
<td>Sensitivity to others’ ambitions ($\eta_{i,A}^j &gt; 0$)</td>
</tr>
<tr>
<td>Positive SVO angle</td>
<td>$\eta_{i,N}^j &gt; 0$ or $\eta_{i,A}^j &gt; 0$</td>
</tr>
<tr>
<td>Conditional cooperation in linear PG</td>
<td>$\eta_{i,N}^j &gt; 0$ or $\eta_{i,A}^j &gt; 0$</td>
</tr>
</tbody>
</table>

ambitions is necessary to explain inequality aversion, in particular among friends and family as observed by Loewenstein et al. (1989).

Through the connection to prospect theory our model can also reduce other-regarding preferences to physiological processes that control the decision-maker’s sensitivity to her reference points. Evolutionary roots of these processes seem more than likely (e.g., Harman, 2010, on the evolutionary roots of altruism). However, other-regarding reference-dependent preferences with their different components appear to be too complex to have evolved all at once. We instead imagine the following evolutionary stages: First, animals that are aware of their own needs to guarantee their survival have an advantage over animals that starve if they do not realize that their food sources run out. This gives loss aversion. Second, being aware of one’s social status provides an advantage in the competition for potential mates. Among prey animals, it is also advantageous to be faster or more intelligent than other animals in the herd in order to avoid predators. Thus the evolutionary benefit of self-centered social comparisons. Empathy requires a society or family of some sorts, like a pack of wolves, or a clan of monkeys. In this case, a hierarchical structure provides the leader with advantages (like first access to food sources), but also responsibilities (like defending the group against intruders). The leader must guarantee not just his own survival, but that of the entire group. Empathy with others’ needs is clearly advantageous in this respect. Finally, empathy with others’ ambitions fits well with the observation that some animal mothers defend their offspring even at the cost of their own lives. Other-regarding preferences in humans are accordingly also determined to a large extent by whom the decision-maker includes in her definition of “family”: relatives, friends, neighbors, or even members of the same country or culture.

Even a competitive decision-maker may find it advantageous to cater to the needs of others, if these people are able to support her ambitions in turn. The motivation to do better than others increases a group’s productivity and thus may benefit all. However, such ambitious persons can also cause social injustice in hierarchical societies, in which the best opportunities to become rich and powerful are limited to only a few individuals. For example, a political leader might convincingly act in the interest of her community, but only until all of her supporters have reached a certain level of wealth. Any additional increase in social welfare is considered fair game (and preferably should end up in the politician’s hands).

Our model is limited in that it only applies to static decisions. In repeated games or
games with sequential decisions, previous outcomes might establish other reference points that are much more salient than references to needs or ambitions. However, we are confident that future research can extend our model to acts of reciprocity (cf. Charness and Rabin, 2002; Falk and Fischbacher, 2006) or aspiration-based heuristics such as win-stay lose-shift (e.g. Chasparis et al., 2013). The ENA model might also provide a mathematical foundation for identity economics (Akerlof and Kranton, 2000), possibly in combination with models of spatial discounting (Perrings and Hannon, 2001) to quantify social distance. Finally, it is straightforward to define a social welfare function that either aggregates individual gain-loss utilities and related sensitivity parameters or possibly even treats the social planner as a person with own needs and ambitions.

Another possible limitation of our model is its relatively large number of parameters, i.e. degrees of freedom. Although many of these parameters can be determined independently of each other given a neutral context, this might consume too many resources in an experimental application of the model to be worthwhile. For the same reason, the model is difficult to use when developing a post-hoc explanation for an experimental observation, unless the experimental subjects are available for additional tests. However, inequality aversion, as a special case of our model, can produce a rough estimate of the players’ preferences, if the researcher suspects empathy for others’ ambitions to have played a role in the experiment. If these suspicions are confirmed, more elaborate tests using the empathy model can help fine-tune the preference estimate. Similarly, standard experimental procedures concerning the elicitation of risk and loss preferences or social value orientation might already provide enough information to decide if subject behavior is influenced by reference points.

Yet the value of a theoretical model does not lie solely in its explanatory power with respect to known empirical phenomena, but also in its capability of making “novel” predictions (Lakatos, 1970). We set out to reduce inequality aversion to prospect theory and concluded that the Fehr-Schmidt parameters must be positively correlated to achieve this result. Similarly, monotonocity assumptions led us to conclude that the decision-maker’s sensitivity to others’ ambitions may be negatively correlated with her extent of loss aversion. And in the attempt to explain hump-shaped preferences in linear public goods games, we discovered a new, v-shaped, preference type located between conditional cooperators and altruists. The ENA model likely yields other unexpected results in additional applications. We therefore encourage future research to address other experimental and theoretical findings on other-regarding preferences from the perspective of the ENA model, especially if related to decisions under risk or uncertainty.

A Proofs

Proof of Proposition 4.2

Proof. We first prove that 2. implies 1.

If \( u^{ENA}(x|r) \) satisfies M then

\[
u^{ENA}(x_i + \delta, x_{-i}|r) > u^{ENA}(x|r) \tag{A.1} \]

or (because other persons’ needs are irrelevant in this comparison)
\[
\delta + \eta_{i,N}^j [\mu_i(x_i + \delta|r_i) - \mu_i(x_i|r_i)] \\
+ \sum_{j \neq i} \eta_{i,A}^j (\mu_i(x_i + \delta|x_j) - \mu_i(x_i|x_j)) + \eta_{i,A}^j (\mu_i(x_j|x_i + \delta) - \mu_i(x_j|x_i))] > 0. \tag{A.2}
\]

Condition \([A.2]\) holds for every reference point \(r_i\) and for all \(x \in X\). There are three possible cases given the ordering of \(r_i, x_i, \) and \(x_i + \delta\). However, \([A.2]\) is the most difficult to satisfy if \(x_i \geq r_i\), so that we can ignore the other two cases. More possible cases arise given the ordering of \(x_i, x_i + \delta, \) and the \(x_j\). Again we only need to consider one extreme case, for which \(x_i \geq x_j\) for all \(j\). Assuming \(A3'\), \([A.2]\) then implies

\[
\delta (1 + \eta_{i,N}^j + \sum_{j \neq i} [\eta_{i,A}^j - \lambda \eta_{i,A}^j]) > 0. \tag{A.3}
\]

For \(\delta\) this gives the first statement. For the reverse implication note that, by \([A.3]\), \([A.2]\) is satisfied for the case where \(x_i \geq r_i\) and \(x_i \geq x_j\) for all \(j\). However, it is easily verified that \([A.3]\) also implies similar conditions which satisfy \([A.2]\) for the other possible cases. And if \([A.2]\) holds in all cases, \(u^{ENA}(x|r)\) satisfies \(M\). \(\Box\)

**Proof of Theorem 5.1**

**Proof.** We can apply \(A3'\) and \(\eta_{i,N}^j = 0\) to \(u^{ENA}_i(x|r)\) to obtain

\[
u_i(x) = x_i + \eta_{i,A}^j \sum_{j \neq i} (\max\{x_i - x_j, 0\} - \lambda_i \max\{x_j - x_i, 0\})
\]

\[+ \sum_{j \neq i} \eta_{i,A}^j (\max\{x_j - x_i, 0\} - \lambda_i \max\{x_i - x_j, 0\}) \tag{A.4}\]

or

\[
u_i(x) = x_i + \sum_{j \neq i} (\eta_{i,A}^j - \lambda_i \eta_{i,A}^j) \max\{x_j - x_i, 0\}
\]

\[+ \sum_{j \neq i} (\eta_{i,A}^j - \lambda_i \eta_{i,A}^j) \max\{x_i - x_j, 0\}. \tag{A.5}\]

The Fehr-Schmidt model follows as a special case after substituting \(-\alpha_{ij}/n-1 \equiv \eta_{i,A}^j - \lambda_i \eta_{i,A}^j\) as well as \(-\beta_{ij}/n-1 \equiv \eta_{i,A}^j - \lambda_i \eta_{i,A}^j\). This yields \(\alpha_{ij}\) and \(\beta_{ij}\) as functions of \(n\):

\[
\alpha_{ij}(n) = (n - 1)(\lambda_i \eta_{i,A}^j(n) - \eta_{i,A}^j(n)) \tag{A.6}
\]

\[
\beta_{ij}(n) = (n - 1)(\lambda_i \eta_{i,A}^j(n) - \eta_{i,A}^j(n)). \tag{A.7}
\]

By letting \(\eta_{i,A}^j \equiv (n-1)\eta_{i,A}^j(n) \geq 0\) for all \(j\) (including \(j = i\)), we further obtain parameter equations that are independent of group size \(n\):

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\[ \alpha_{ij} = \lambda_i \hat{\eta}^i_{k,A} - \hat{\eta}^j_{k,A} \]  
(A.8)

\[ \beta_{ij} = \lambda_i \hat{\eta}^j_{k,A} - \hat{\eta}^i_{k,A}. \]  
(A.9)

Finally, given \( \hat{\eta}^j_{k,A} > 0 \), we can use these two equations to express \( \alpha_{ij} \) as a function of \( \beta_{ij} \) by substituting \( \lambda_i \), which is context-independent (or at least more stable than \( \hat{\eta}^j_{k,A} \) and \( \hat{\eta}^j_{k,A} \)):

\[ \alpha_{ij} = \frac{\hat{\eta}^i_{k,A} \beta_{ij} + (\hat{\eta}^i_{k,A})^2 - (\hat{\eta}^j_{k,A})^2}{\hat{\eta}^2_{k,A}} \]  
(A.10)

This shows that \( \alpha_{ij} \) and \( \beta_{ij} \) are positively correlated within the same context. \( \square \)

**Proof of Proposition 6.3**

**Proof.** Let \( \phi(\lambda_i) \equiv \frac{b_j(\lambda_i)}{a(\lambda_i)} \). Then, because \( \arctan(x) \) is strictly increasing:

\[ \frac{\partial}{\partial \lambda_i} \mathcal{L}^j_{i,A} \equiv 0 \Leftrightarrow \frac{\partial}{\partial \lambda_i} \phi(\lambda_i) \equiv 0 \]  
(A.11)

With \( a(\lambda_i) = 1 - \sum_{k \neq i} b_k(\lambda_i) \) we have

\[ \frac{\partial}{\partial \lambda_i} \phi(\lambda_i) = \frac{[1 - \sum_{k \neq i} b_k(\lambda_i)]b'_j(\lambda_i) + b_j(\lambda_i) \sum_{k \neq i} b'_k(\lambda_i)}{[1 - \sum_{k \neq i} b_k(\lambda_i)]^2}. \]  
(A.12)

This implies

\[ \frac{\partial}{\partial \lambda_i} \phi(\lambda_i) \equiv 0 \Leftrightarrow b'_j(\lambda_i) + \sum_{k \neq i} [b_j(\lambda_i) b'_k(\lambda_i) - b_k(\lambda_i) b'_j(\lambda_i)] \equiv 0. \]  
(A.13)

We now distinguish three cases depending on whether \( x_i \) is larger, equal to, or smaller than \( x_j \).

1. \( x_i = x_j \). Here \( b_j(\lambda_i) = 0 \) is independent of \( \lambda_i \) so that also \( b'_j(\lambda_i) = 0 \). This immediately gives

\[ \frac{\partial}{\partial \lambda_i} \mathcal{L}^j_{i,A} = 0 \]  
(A.14)

For the other two cases we also need to distinguish \( x_i > x_k \) and \( x_i < x_k \) to determine \( b_k(\lambda_i) \) and \( b'_k(\lambda_i) \) for each other person \( k \)\(^{44}\). If \( x_i > x_k \), then \( \lambda_i b'_k(\lambda_i) - b_k(\lambda_i) = \eta^j_{k,A}(n) \) and \( b'_k(\lambda_i) = \eta^k_{i,A}(n). \) If \( x_i < x_k \), then \( \lambda_i b'_k(\lambda_i) - b_k(\lambda_i) = -\eta^k_{i,A}(n) \) and \( b'_k(\lambda_i) = -\eta^i_{i,A}(n). \)

2. \( x_i > x_j \). Here \( b_j(\lambda_i) = \lambda_i \eta^j_{i,A}(n) - \eta^j_{i,A}(n) \) and \( b'_j(\lambda_i) = \eta^j_{i,A}(n). \) From (A.13) we obtain

\[ \eta^j_{i,A}(n) + \sum_{k \neq i} [\eta^j_{i,A}(n)(\lambda_i b'_k(\lambda_i) - b_k(\lambda_i)) - \eta^k_{i,A}(n) b'_j(\lambda_i)] > 0. \]  
(A.15)

To see that (A.15) indeed holds, we examine for each \( k \) the term

\(^{44}\)If \( x_i = x_k \), then this person can be ignored in the following.
\[ \psi_k \equiv \eta^j_{i,A}(n)(\lambda_i b'_k(\lambda_i) - b_k(\lambda_i)) - \eta^i_{i,A}(n)b'_k(\lambda_i) \]  

(A.16)

If \( x_i > x_k \) then

\[ \psi_k = \eta^j_{i,A}(n)\eta^i_{i,A}(n) - \eta^i_{i,A}(n)\eta^k_{i,A}(n) = \eta^i_{i,A}(n)(\eta^j_{i,A}(n) - \eta^k_{i,A}(n)) \approx 0 \]  

(A.17)

because \( \eta^j_{i,A}(n) \approx \eta^k_{i,A}(n) \). If instead \( x_i < x_k \) then

\[ \psi_k = -\eta^j_{i,A}(n)\eta^k_{i,A}(n) + (\eta^i_{i,A}(n))^2 > 0 \]  

(A.18)

because \( \eta^i_{i,A}(n) > \eta^j_{i,A}(n) \) for all \( j \). (A.15) further implies

\[ \frac{\partial}{\partial \lambda_i} \varphi^j_{i,A} > 0 \]  

(A.19)

3. \( x_i < x_j \). Here

\begin{align*}
  b_j(\lambda_i) &= \eta^j_{i,A}(n) - \lambda_i \eta^i_{i,A}(n) \\
  b'_j(\lambda_i) &= -\eta^i_{i,A}(n)
\end{align*}

(A.20)

(A.21)

From (A.13) we obtain

\[ -\eta^i_{i,A}(n) - \sum_{k \neq i} [\eta^i_{i,A}(n)(\lambda_i b'_k(\lambda_i) - b_k(\lambda_i)) - \eta^j_{i,A}(n)b'_k(\lambda_i)] < 0 \]  

(A.22)

To see that (A.22) indeed holds, we examine for each \( k \) the term

\[ \hat{\psi}_k \equiv \eta^i_{i,A}(n)(\lambda_i b'_k(\lambda_i) - b_k(\lambda_i)) - \eta^i_{i,A}(n)b'_k(\lambda_i) \]  

(A.23)

If \( x_i > x_k \) then

\[ \hat{\psi}_k = (\eta^i_{i,A}(n))^2 - \eta^i_{i,A}(n)\eta^k_{i,A}(n) > 0 \]  

(A.24)

because \( \eta^i_{i,A}(n) > \eta^j_{i,A}(n) \) for all \( j \). If instead \( x_i < x_k \) then

\[ \hat{\psi}_k = -\eta^i_{i,A}(n)\eta^k_{i,A}(n) + \eta^i_{i,A}(n)\eta^j_{i,A}(n) = \eta^i_{i,A}(n)(\eta^k_{i,A}(n) - \eta^j_{i,A}(n)) \approx 0 \]  

(A.25)

because \( \eta^i_{i,A}(n) \approx \eta^k_{i,A}(n) \). (A.22) further implies

\[ \frac{\partial}{\partial \lambda_i} \varphi^j_{i,A} < 0. \]  

(A.26)

\[ \square \]
References


Stahl, D.O., Haruvy, E., 2006. Other-regarding preferences: Egalitarian warm glow,


