Motivated Prospects of Upward Mobility

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Abstract

The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect to move upward on the income ladder and fear that high redistribution may negatively affect them in the future. This paper explicitly models the beliefs agents have about their future income and examines how and when these beliefs are overly optimistic resulting in low redistribution. Agents collectively choose a linear tax rate under uncertainty about their exogeneous future incomes. In addition to the utility from consumption, agents derive utility from the anticipation of their future consumption. This incentivizes them to distort their beliefs. Given the cognitive technology for belief distortion, the motivated prospects of upward mobility emerge endogenously as a result of agents' choices between anticipation and consumption.

1 Introduction

The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect to move upward on the income ladder and fear that the higher redistribution may negatively affect them in the future. This work attempts to formalize the POUM hypothesis by explicitly modeling the voters' beliefs about their prospective incomes. Under certain conditions, enough of the poor believe that they will be rich in the future and the electorate chooses low redistribution.

Previously, the POUM hypothesis has been formalized by Bénabou and Ok (2001). They show that under favorable income dynamics, it is possible that more than half of the voters have an above average expected future income. As a result, more than half of the voters prefer low distribution and vote accordingly. While, according to empirical evidence, both perceived upward mobility (Ravallion & Lokshin, 2000; Cojocaru, 2014) and actual upward mobility (Alesina & La Ferrara, 2005; Alesina & Giuliano, 2011;

Checchi & Filippin, 2004; Bénabou & Ok, 2001) seem to decrease voters' demand for redistribution, it also seems that perceived mobility and actual mobility do not necessarily correlate (Fischer, 2009; Alesina, Glaeser, & Sacerdote, 2001; Gottschalk & Spolaore, 2002). The puzzle then, and what the model in Bénabou and Ok (2001) fails to explain is why prospects of upward mobility decrease the demand for redistribution even in the absence of actual upward mobility. For instance, in the US, the perceived upward mobility is higher than in Europe, producing a higher POUM effect while there does not seem to be much difference in actual upward mobility across the Atlantic (Alesina et al., 2001; Gottschalk & Spolaore, 2002). In addition, as noted by Alesina and Giuliano (2011) and Minozzi (2013), the assumptions underlying the model of Bénabou and Ok (2001) are restrictive and empirically implausible. Therefore, Alesina and Giuliano (2011) suggests that a more plausible mechanism for the POUM effect could be over-optimism and this suggestion is supported by a vast literature in experimental psychology on overconfidence (Alicke & Govorun, 2005; Moore & Healy, 2008; Weinstein, 1980).[1]

A formalization of the POUM hypothesis, which lets voters have overly optimistic beliefs about their future incomes, is provided by Minozzi (2013). In Minozzi's model, citizens vote on future redistribution under uncertainty over their future incomes. When expecting their future consumption, they enjoy anticipation and this incentivizes them to hold optimistic beliefs. The weakness of this model is, however, in its naive technology of belief distortion, which allows citizens to effectively decide what to believe and leaves them with no doubts of whether their beliefs truly represent the reality. This might be too simplistic an assumption and potentially misses important mechanisms of belief distortion as argued by Bénabou and Tirole (2002).

The present work attempts to address these problems in the previously proposed models. The basic structure of our model is similar to Minozzi's (2013) model: When voting for a tax rate according to which the future incomes will be redistributed, agents have uncertainty over their future incomes. After voting, and before the realization and redistribution of their incomes, they anticipate their future consumption. This anticipation creates an incentive to form overly optimistic beliefs. The departure of the current work from Minozzi's (2013) model is most notably in the technology that agents use to distort their beliefs. The cognitive technology for belief distortion in the current work is adopted and adapted from Bénabou and Tirole (2002) and generalized such that we are able to

[1]See also references in Weinberg (2009).

analyze a whole continuum of cognitive technologies varying in the constraints they impose on belief distortion. The conditions for the POUM effect are derived for each of these cognitive technologies, and it is shown that for a set of cognitive technologies the poor prefer optimism and low taxes over realism and high taxes. Also, it is demonstrated how the results of Minozzi's (2013) model are not robust to a bayesian rational updating of beliefs. Furthermore, in addition to strategic belief formation and voting, we consider sincere belief formation and voting as well, and show that when the voters do not think that their beliefs and voting have a significant effect on the tax policy, they always indulge in optimism and may end up making nonoptimal decisions for themselves.

The rest of the work is organized as follows. In section 2, we briefly position the current work into the existing literature in political economy and psychological economics. Section 3 presents the model and derives the conditions for the POUM effect. Also, Minozzi's POUM model is derived as a special case, and its shortcomings are addressed. Section 4 extends the analysis of the model by studying the comparative statistics of changes in the underlying income distribution, presents some welfare analysis and considers the case of nonstrategic belief formation and voting. Section 5 concludes. All proofs of the lemmas and propositions are collected in the appendix.

2 Relations to the literature

2.1 Political Economy and Redistribution

If the rational choice model with narrowly defined utility together with the Median Voter Theorem cannot be corroborated by empirical observations, one of these underlying assumptions, rational choice or median voter's power, must be wrong. It might either be the case that modeling voters as income maximizing agents does not capture all the relevant aspects of their decision-making or that the outcome that the electoral system provides does not reflect the preferences of the median voter.[2]

In this work, the policy outcome is assumed to be the median voter's bliss point and the focus, therefore, is on the former of these possible caveats. Hence, this work can be positioned into the strand of literature initiated by Romer (1974) and Meltzer and

[2]Reasons for the latter could be, for instance, unequal political participation (Bénabou, 2000; Mahler, 2008), the political influence of the rich (Gilens, 2005), campaign contributions (Karabarbounis, 2011), economic inequality (Lupu & Pontusson, 2011; Solt, 2008), electoral systems (Iversen & Soskice, 2006; Cukierman & Spiegel, 2003; Austen-Smith, 2000), and interest groups (Dixit & Londregan, 1998).

Richard (1981), which aims to explain the extent of redistribution in democratic societies by studying what determines the voters' demand for redistributive policies. To ensure the existence of political equilibrium, this literature mostly focuses on unidimensional policy choices, usually choices over a linear tax rate with lump-sum transfers. With this simplification, the policy preferences of voters are single-crossing, and the median voter theorem applies. The remaining question then, and the interest of this literature is how does the median voter decide on her vote.

The obvious starting point is the voter's current income, but preferences so narrowly defined have been unsatisfactory in explaining real-world tax policies (Bénabou, 1996; Borck, 2007; Luebker, 2014). Other factors explaining the demand for redistribution proposed in this literature are, for instance, efficiency costs of taxation (Meltzer & Richard, 1981), different individual (Piketty, 1995) and cultural (Corneo & Grüner, 2002; Alesina, Glaeser, & Glaeser, 2004) histories and experiences, social preferences, such as altruism, inequality aversion and fairness considerations (Alesina & Angeletos, 2005; Alesina, Cozzi, & Mantovan, 2012; Alesina et al., 2004; Fong, 2001), structure and organization of the family (Todd, 1985; Esping-Andersen, 1999; Alesina & Giuliano, 2010), and social mobility (Piketty, 1995; Hirschman & Rothschild, 1973; Bénabou & Ok, 2001).[3] In addition to increasing the scope of preferences, the literature has also studied the role of beliefs (Piketty 1995, Alesina and Angeletos, 2005a) and biased beliefs (Minozzi, 2013; Bénabou and Tirole, 2006; Bénabou, 2008). Given this rich set of explanations for the extent of redistribution, a parsimonious model seems unlikely, and a single factor should be interpreted as a part of the story, complementing and rivaling the other explanations. The part of the story we focus from now on in this work is the POUM effect.

First, social mobility, broadly speaking, refers to both upward and downward mobility. The premise is that instead of current income, the policy preferences depend on future income. When voters are worried that their incomes might decrease relative to others, they could use redistribution as insurance against downward mobility. This would increase the demand for redistribution. The POUM, on the other hand, focuses on the possibility of upward mobility, which has the opposite effect: When the voters expect their incomes to increase relative to others, they vote for less redistribution.

However, social mobility is also often connected to the roles of chance, circumstances, and effort in determining income. If voters perceive that the effort one exerts determines

[3]A review on the preferences for redistribution is provided by Alesina and Giuliano (2011).

one's prospects, then they can believe in a mobile society, but if they believe that the circumstances have a major role in determining one's prospects, then they believe in immobile society. Piketty (1995) studies how the interaction of social mobility and beliefs about determinants of income affects voting. In the present work, incomes are exogenous and, in the spirit of the POUM hypothesis, beliefs about social mobility refer solely to beliefs about the levels of future incomes.

The first characterization of the POUM effect is perhaps Hirschman's (1973) "tunnel effect" in which people's demand for redistribution decreases when they see the incomes of relatable people in their environment increase. They expect that their turn will follow soon and they, therefore, tolerate more inequality.

The first formalization of the POUM effect was provided by Bénabou and Ok (2001). Their approach is to maintain rational expectations and show that favorable income dynamics can make more than half of the voters to expect above-average incomes. The agents vote for a redistribution policy, which will be in place for a predetermined time, and expect their incomes to evolve according to a stochastic transition function. The deterministic part of this transition function is concave, which allows a majority of voters to believe that they will receive an above average income in the future. The stochastic part consists of skewed income shocks, which ensure that the skewness of the original income distribution is preserved. The combination of skewed shocks and concave prospects lets the expected incomes and realized incomes diverge and makes the POUM effect possible with invariant income distribution and rational expectations.

Minozzi (2013) develops an "Endogenous Beliefs Model" and proposes an explanation for the POUM effect by abandoning rational expectations and letting voters form overly optimistic prospects about their future income. Minozzi's model relies on a game theoretic multi-self approach, where each citizen has, without their knowledge, an "agent" who controls their beliefs and optimizes the trade-off between optimistic beliefs and nonoptimal actions. Citizens receive an anticipatory flow utility in period 1 and a flow utility called *outcome utility* in period 2, when they receive their stochastic and exogenous incomes. The agent's objective function for belief formation consists of these two sources of utility. In choosing the optimal beliefs by solving the trade-off between anticipatory and outcome utility, the agent knows the prior prospects of the citizen and how the tax policy is dependent on the chosen beliefs. If the poor citizens value anticipation enough, they will end up with optimistic beliefs and vote for low redistribution.

The POUM effect also emerges in the model of Bénabou and Tirole (2006). In their model, agents have overly optimistic beliefs about their productive ability and, hence, future income. When they believe themselves to be abler than others, they prefer less redistribution. Although their model, as the present work, derives the POUM effect by letting agents hold overly optimistic beliefs, their work differs from the current one in its mechanism for the belief distortion. Specifically, what incentivizes the agents to hold biased beliefs differs. In their work, agents suffer from deficient willpower and form overly optimistic beliefs about their abilities in order to motivate themselves and in this way to compensate for the imperfect willpower. That is, belief distortion works as a commitment device. In current work, on the other hand, the beliefs are distorted since beliefs can be consumed and overly optimistic beliefs bring higher anticipatory utility. However, these different incentives are not mutually exclusive, and probably both are at work. The explanation for the POUM effect in Bénabou and Tirole (2006) should, therefore, be seen as complementary to the current work.

2.2 Psychological Economics and Motivated Beliefs

Psychological economics attempts to draw inspiration from the field of psychology and build models that better represent the cognitive processes of decision makers aiming to close the apparent gap between the observed behavior of people and the behavior postulated by the rational choice theory. The rational choice theory is, however, the primary method of analysis in economics and the work in psychological economics, rather than abandoning this theory, proceeds by widening its scope.[4] The current work broadens the rational choice theory to accommodate psychological factors in two ways. First, we widen the scope of preferences to include anticipation of future consumption. Second, we let agents make optimal decisions about their beliefs.

Anticipatory utility is perhaps little used but certainly not a new idea in the literature of economics: "*When calculating the rate at which future benefit is discounted, we must be careful to make allowance for the pleasures of expectation*", writes Alfred Marshall in his *Principles of Economics* published in 1891 (p. 178, quoted in Löwenstein (1987)). Our mind is both an information processing machine by which we make our decisions and a consuming organ deriving satisfaction from our emotions, as Schelling (1987) put it. That is, we use our beliefs to predict the consequences of our actions, but we also

[4]On psychological economics, see, for instance, Rabin (2002) and Tirole (2002).

consume them. Due to this latter function of beliefs, we derive utility or incur disutility simply by believing certain things. As experiments have shown, this consumption value of beliefs has consequences for our information processing (Kunda, 1990; Averill & Rosenn, 1972; Lerman et al., 1998) and our behavior (Cook & Barnes Jr, 1964; Löwenstein, 1987).

Anticipatory utility is modeled usually by letting the utility function have a term which is a linear (Minozzi, 2013; Bénabou, 2008, 2012; Brunnermeier & Parker, 2005) or a general (Caplin & Leahy, 2001; Köszegi, 2010; Bernheim & Thomadsen, 2005) function of expectation of a later period utility flow. In Akerlof and Dickens (1982), agents incur psychic costs of fear modeled as a "fear cost function" which depends on the perceived probability of an accident in their hazardous job.

In addition to preferences, an important element of decisions in an uncertain world is beliefs. Hence, to understand decisions, it is crucial to understand beliefs. The departure from rational expectations is motivated by vast literature in psychology (Alicke & Govorun, 2005; Moore & Healy, 2008; Weinstein, 1980) and behavioral economics (De Bondt & Thaler, 1995; Skala, 2008). In addition to challenging the objectivity of beliefs, the literature in psychology directs us towards alternative options: Biases in beliefs are not random but they rather seem to be incentivized and partly determined by desires (Kunda, 1990; Braman & Nelson, 2007; Redlawsk, 2002; Taber & Lodge, 2006). This literature of motivated reasoning asserts that human information processing, memories, and beliefs are affected by our motivations. In addition to *accuracy goals*, reasoning can be motivated by *directional goals*, that is, by desires and preferences.

The literature on motivated reasoning has inspired models of biased beliefs where the beliefs are a result of optimizing the trade-off between accuracy goals and directional goals. Anticipatory utility is one way to model such a directional goal for reasoning, but a complete model also requires the means for belief distortion. We call a *cognitive technology* a framework which provides the agents with the ways and constraints of distorting their beliefs. There are roughly two kinds of cognitive technologies used in the literature. In the first of these which we will call *naive cognitive technologies*, the beliefs can be simply chosen, and they do not need to depend on the prior beliefs or the objective probability distributions of reality. For instance, Minozzi (2013), Brunnermeier and Parker (2005), and Akerlof and Dickens (1982) use a naive cognitive technology. We call the second kind of cognitive technology a *sophisticated cognitive technology.* If the cognitive technology is sophisticated, agents realize that they have incentives to bias their beliefs and assess their beliefs accordingly. Also, the emerging beliefs are influenced by the prior beliefs and are anchored in reality. This second type of cognitive technology is used in Bénabou and Tirole (2002), Bénabou and Tirole (2006), Bénabou (2008), Bénabou (2012), and Kopczuk and Slemrod (2005), and reviewed in Bénabou (2015) and Bénabou and Tirole (2016). The names for these two types of cognitive technologies follow from their different assumptions on the agents' degree of Bayesian sophistication.

Minozzi (2013) calls the nonstandard beliefs that emerge in his model *endogenous* beliefs whereas Bénabou (2015) refer to these beliefs as *motivated* beliefs.[5] In this work, these terms are used interchangeably. However, the term *motivated* beliefs is more informative. After all, all beliefs that are determined within a model, can be called endogenous. For instance, in this sense, the usual rational expectations are endogenous beliefs as well.

To sum up, a model containing belief distortion has two crucial elements. First, agents must have an incentive to hold biased beliefs. Using the language of Bénabou and Tirole (2002), this can be called the demand for distorted beliefs. In the current work, agents are incentivized to have biased beliefs by letting them derive utility from their high hopes. Second, agents must be able to influence their beliefs. This can be called the supply of distorted beliefs. The supply of distorted beliefs depends on the cognitive technology which sets the possibilities and limits for belief distortion. The current work considers the whole continuum of cognitive technologies from the completely naive to the fully sophisticated. Given the incentives and the technology of belief formation, biased subjective beliefs emerge as a result of optimization. This optimization involves trading-off the benefits of holding biased beliefs against the costs of inferior decisions due to inaccurate information and is subject to the constraints of the cognitive technology. The emergence of non-standard beliefs as a result of optimization and purposeful actions distinguishes the motivated beliefs framework from the mechanical failures of rationality or bounded rationality, which leave the motivations of actions intact and only impose constraints on reasoning (Bénabou & Tirole, 2016).

[5] Brunnermeier and Parker (2005) call them optimal beliefs.

Period 0	Period 1	Period 2
Receive signals σ	Recall $\hat{\sigma}$ and form beliefs	Incomes realize
		Redistribution
Choose λ	Vote for redistribution	Consumption
	Anticipation	

Figure 1: Timeline

3 The Model

3.1 The Economy and the Timing of the Model

The economy consists of a unitary continuum $i \in [0, 1]$ of risk-neutral agents who collectively decide on an income tax policy under uncertainty about their exogenous future incomes. In period 0, agents receive a signal conveying information about their prospective future incomes. In period 0, they also engage in various conscious and unconscious psychological processes of belief distortion, reality denial, and information avoidance which determine the signal they will remember in period 1.[6] In the beginning of period 1, agents recall a signal and form beliefs about their future incomes based on their recollection. Then they vote for redistribution. They get to know the policy outcome immediately after the vote, and in the rest of period 1 they experience anticipatory utility as they anticipate their consumption which occurs in period 2, right after the incomes have been realized and redistributed. The timeline is given in Figure 1.

3.2 Information and Beliefs

In period 0, each agent receives a noisy signal $\sigma_i \in \mathcal{F} = \{F_L, F_H\}$ conveying information about their future incomes. These signals are identical and independent draws from the

[6] Agents have imperfect recall in the sense that they forget information. The underlying game theoretical construct to model this inconsistency is to model agents consisting of two players, their two temporal selves (see Bénabou and Tirole (2002)). Also, the parallel interpretation throughout the paper is that the parents have influence over what their offsprings belief when the offsprings are making voting decisions.

following probability mass function:

$$g(\sigma) = \begin{cases} q & \text{if} \quad \sigma = F_H \\ 1-q & \text{if} \quad \sigma = F_L \end{cases}, \tag{1}$$

where F_H and F_L are probability distributions over the future income levels such that $\int_y y dF_H(y) > \int_y y dF_L(y)$ and $y \geq 0$.[7] Using the language of Minozzi (2013), we call the agents who receive signal $\sigma = F_H$ the likely rich and the agents who receive signal $\sigma = F_L$ the likely poor. With a large number of agents, a fraction q of the population is likely rich and a fraction $1-q$ likely poor. Furthermore, we assume that the likely poor agents constitute a majority, that is, we assume $q < \frac{1}{2}$. As agents are risk-neutral, a sufficient statistics for the analysis are the means of the distributions F_H and F_L: $y_H = \int_y y dF_H(y)$ and $y_L = \int_y y dF_L(y)$, the incomes that the likely rich and the likely poor, respectively, expect to earn in period 2. In the following, we refer to these distributions by their means and let the signal set be $\{y_L, y_H\}$.[8]

The possibility for belief distortion arises in the period 0 actions. After receiving a signal, each agent decides which of the two signals she will recall in period 1. As we will see, a likely poor agent has an incentive not to recall her true prospects. On the other hand, we make a sensible assumption that the likely rich agents will always choose to remember the signal they received and they, therefore, have no interesting decision to analyze. After all, if they underestimate their income, they lose anticipatory utility.[9] Hence, we focus mainly on the more interesting decisions of the likely poor agents. Formally, in period 0,

[7] Here the signals are independent for simplicity and to induce some heterogeneity in the resulting income distribution. In general, the signals may be correlated. The special case of perfectly correlated types and signals can be used if the unknown variable is more common to agents in the sense that it reflects some general workings of the economy, like return to effort as in Bénabou and Tirole (2006), government efficiency as in Bénabou (2008) or expected value of a joint project as in Bénabou (2012).

[8] We use a simplifying shortcut here. The underlying formal process, of course, is that Nature draws a state of the world, which determines the incomes of each agent. Agents receive some information about the state of the world via a signal determined by a signal function which lets them know a set of states of the world. Using the prior belief and the signal they then form a posterior belief. The posterior belief is, therefore, a function of the signal and fixed prior beliefs, so it is straightforward to associate a signal with a posterior belief and let the outputs of the signal function be the posterior beliefs agents have immediately after receiving the signal. Moreover, as the signal is a deterministic function of the state of the world, which Nature draws, we can simply let the received signal have the given distribution.

[9] This seems a very plausible conjecture but technically this is not that simple. Depending on the off-equilibrium path beliefs, an agent sending a low signal might end up with higher beliefs than when sending a high signal. In the appendix, we make an assumption about these off-equilibrium path beliefs to exclude this peculiar theoretical possibility.

a likely poor agent i chooses a recall rate $\lambda_i \in [0, 1]$ defined as

$$\lambda_i \equiv Pr[\hat{\sigma}_i = y_L | \sigma_i = y_L], \tag{2}$$

where $\hat{\sigma}_i$ denotes both the signal agent i recalls in period 1 and the action she chooses in period 0.[10]

In period 1, agent i's information is based on a recalled signal $\hat{\sigma}_i \in \{y_L, y_H\}$. The memory of agents is probabilistic and their actions in period 0 determine the probability of each recollection. With probability λ_i, a likely poor agent will correctly recall $\hat{\sigma}_i = y_L$ and with probability $1 - \lambda_i$, she will recall $\hat{\sigma}_i = y_H$. By assumption, the likely rich agents always recall $\hat{\sigma}_i = y_H$. Of course, we are not claiming that people literally choose exact probabilities for the occurrences of their future memories. The choices in period 0 should be interpreted as all sorts of unconscious and conscious processes and actions that affect the availability of certain recollections. In equilibrium, agents act as if they were choosing optimal recall rates.

However, agents may not be completely in control of their beliefs. They may know that they have a tendency to forget bad news and remember good news. Therefore, they may not fully trust their recollections. If an agent i recalls $\hat{\sigma}_i = y_H$ in the second period, she will assign a reliability $r(\lambda_i)$ to this signal:

$$r(\lambda_i | \chi) = Pr[\sigma_i = y_H | \hat{\sigma}_i = y_H] = \frac{q}{q + \chi(1-q)(1-\lambda_i)}, \tag{3}$$

where λ_i is given by the period 0 strategy of agent i. χ is the naivete parameter measuring the degree of Bayesian sophistication. $\chi = 1$ corresponds to the full Bayesian rationality which is usually assumed in the applications of game theory.[11] In the other extreme, $\chi = 0$, and the reliability of received signal is always 1. This means that in period 1, agents will completely trust their recollections and that in period 0, they are completely in control of their beliefs in period 1. The role of χ will be analyzed extensively later. Note that the reliability in (3) is defined only for the signal $\hat{\sigma}_i = y_H$. By assumption, only the likely poor might send a signal $\hat{\sigma} = y_L$, so the reliability of this signal is always 1.

With probability $1 - \lambda_i$, a likely poor agent recalls $\hat{\sigma}_i = y_H$ and is an *optimist*. In

[10]In the jargon of game theory, in period 0, an agent i plays a mixed strategy $\begin{pmatrix} y_L & y_H \\ \lambda_i & 1-\lambda_i \end{pmatrix}$.

[11]Bayesian rationality refers to the use of Bayes rule in updating beliefs.

period 1, she expects a gross income

$$E[y_i|F_{1,i}] = r(\lambda_i)y_H + (1 - r(\lambda_i))y_L, \tag{4}$$

which is a linear combination of the expected incomes of the two different types weighted by the reliability. $F_{1,i}$ is the information of agent i in period 1. Note how a decrease in λ_i increases the probability of being an optimist and, as we will see, the expected anticipatory utility. However, the effect is nonlinear for $\chi > 0$ since the reliability decreases as λ_i increases. The more likely it is that a likely poor agent i memorizes a false signal, the less reliable signal $\hat{\sigma}_i = y_H$ becomes. The more agents try to distort their beliefs, the more cautious they are when they are forming their beliefs.

With probability λ_i, a likely poor agent recalls $\hat{\sigma}_i = y_L$ and is a *realist*. As the reliability of signal $\hat{\sigma}_i = y_L$ is always 1, in period 1, she expects a gross income

$$E[y_i|F_{1,i}] = y_L. \tag{5}$$

The likely rich will recall $\hat{\sigma}_i = y_H$, and as they also do not know whether they truly are likely rich or likely poor, their expected income will coincide with the expected income of optimistic likely poor.

3.3 Preferences

In period 2, agents receive an exogenous income, pay taxes, and consume their disposable income. The government's budget is balanced, and all tax revenue collected via a linear income tax is transferred in equal lump-sums to agents. There is no wastage in the redistribution. Agents derive utility linearly from their consumption:

$$u_{2,i}(c_i) = c_i(\sigma_i, \tau) = (1 - \tau)y_i + \tau\bar{y}, \tag{6}$$

where c_i denotes consumption, τ is the income tax rate, and $\bar{y}$ is the average income:

$$\bar{y} = qy_H + (1 - q)y_L. \tag{7}$$

In period 1, agents do not yet know their income, but given their beliefs, they form expectations and experience a flow utility due to anticipation. The intertemporal preferences of agents from the perspective of period 1 are given by

$$u_{1,i}(\hat{\sigma}_i, \tau) = sE[u_{2,i}|F_{1,i}] + \delta E[u_{2,i}|F_{1,i}] = (s+\delta)E[(1-\tau)y_i + \tau\bar{y}|F_{1,i}], \tag{8}$$

where the expectations are conditioned on the period 1 information $F_{1,i}$, $\delta \in [0,1]$ is the standard discount factor and $s \geq 0$ is the "savoring" parameter which measures the importance of anticipation. The anticipatory utility is proportional to agent's expectations. The higher expectations she has, the more utility she derives. This gives agents an incentive to distort their beliefs. Setting $s = 0$ yields the standard case with no anticipatory utility and therefore no incentive to distort beliefs. The discount factor and the savoring parameter are common to all agents.

The intertemporal utility from the period 0 perspective is

$$\begin{aligned} u_{0,i}(\sigma_i, \hat{\sigma}_i, \tau) &= \delta E[sE[u_{2,i}|F_{1,i}]|F_{0,i}] + \delta^2 E[u_{2,i}|F_{0,i}] \\ &= \delta s E[(1-\tau)y_i + \tau\bar{y}|F_{1,i}] + \delta^2 E[(1-\tau)y_i + \tau\bar{y}|F_{0,i}]. \end{aligned} \tag{9}$$

The expected period 1 flow utility depends on the information in period 1 and the expected period 2 flow utility depends on the information in the period 0.[12] That is, in period 0, agents know the true objective expectation of their incomes in period 2, but they also know that they will receive higher utility in the period 1 if their beliefs in period 1 are biased upwards. The trade-off, which the optimal period 0 actions optimize, can be seen clearly here. Agents gain more utility if they have high hopes, but as we will see, with high hopes they will vote for low taxation, which then lowers their consumption in the last period.

3.4 The Polity and Voting Decisions

The agents vote for tax rate $\tau \in [\underline{\tau}, \overline{\tau}]$ in the beginning of period 1. Their policy preferences are given by (8), and they depend on the subjective beliefs they have in period 1.

[12] Note that since information is lost between periods 0 and 1 and $F_{1,i}$ contains less information than $F_{0,i}$ the law of iterated expectations does not hold and $E[sE[u_{2,i}|F_{1,i}]|F_{0,i}] \neq sE[u_{2,i}|F_{0,i}]$, but the smaller information set wins and $E[sE[u_{2,i}|F_{1,i}]|F_{0,i}] = sE[u_{2,i}|F_{1,i}]$.

Maximization with respect to the tax rate leads to the following voting rule:[13]

$$\tau_i^* = \begin{cases} \underline{\tau} & \text{if} \quad E[y_i|F_{1,i}] \geq \bar{y} \\ \overline{\tau} & \text{if} \quad E[y_i|F_{1,i}] < \bar{y} \end{cases}, \tag{10}$$

where τ_i^* is the preferred tax rate of agent i. If an agent expects in period 1 to earn an above average income in the period 2, she will vote for the minimum redistribution, and if she expects to earn a below average income, she will vote for the maximum redistribution. This parallels the classic result of Meltzer and Richard (1981). The linearity of the policy preferences leads to corner solutions, which simplifies the analysis here. In reality, there are, of course, additional considerations that restrict the tax policies between the extremes. As we will see, setting $\overline{\tau} < 1$ and $\underline{\tau} > 0$ allows us to exogenously restrict the set of feasible tax policies.

As the policy preferences given by (8) are single-peaked, the Median Voter Theorem (Black, 1948; Downs, 1957) applies and the tax policy will be the tax rate preferred by the median voter. With two groups of voters, the median voter's opinion will be the opinion of the majority.

If agents could not manipulate their expectations or if they did not have any incentives to distort their beliefs (e.g., $s = 0$), they would vote according to their objective prospects, and the unique equilibrium would be the likely poor voting for high taxes and the likely rich voting for low taxes. The median voter would be among the likely poor, and the policy in the unique equilibrium would be high taxes. We will see how the possibility of subjective beliefs that differ from the objective standard allow additional equilibria with other policy outcomes.

Throughout the analysis, we focus on symmetric decisions within the two groups of voters. All of the likely rich choose $\hat{\sigma} = y_H$ and all of the likely poor choose the same λ. An optimist will always vote for $\tau = \underline{\tau}$ as seen from (10) and (4) and noting that $r(\lambda) \geq q$ for all $\lambda \in [0, 1]$. A realist will always vote for $\tau = \overline{\tau}$ by (5). Also, the likely rich will always vote for $\tau = \underline{\tau}$, similarly to the the optimistic likely poor. Putting all this together, the policy outcome can be derived as a function of λ. The total share of agents

[13] We assume that an indifferent agent votes for low taxes. This assumption turns out to be quite crucial as it determines the tax policy in the low tax equilibrium of the model in the case of $\chi = 1$. We could, however, suppose, that there is an arbitrarily small amount of wastage involved in taxation, or that the voters deviate an arbitrarily small amount from the full Bayesian rationality, which both would solve the indifference for low taxes.

expecting above average income is $q + (1-q)(1-\lambda)$. The policy outcome τ^* depends on whether this share exceeds $\frac{1}{2}$ or not:

$$\tau^* = \begin{cases} \underline{\tau} & \text{if} \quad \lambda < \frac{1}{2(1-q)} \\ \overline{\tau} & \text{if} \quad \lambda \geq \frac{1}{2(1-q)} \end{cases}. \tag{11}$$

In line with Minozzi's (2013) model, we first let the agents vote strategically.[14] That is, they take account that their vote might be pivotal. As will be shown later, if agents voted sincerely, the trivial outcome would be everyone maximizing the anticipatory utility.[15]

3.5 Conditions for the POUM effect, $\tau \in [0, 1]$

To gain some intuition and to analyze an interesting special case, we first set $\overline{\tau} = 1$ and $\underline{\tau} = 0$. The more general and more realistic case of $\overline{\tau} < 1$ and $\underline{\tau} > 0$ is analyzed in the next section.

Now that we know the voting decisions in period 1, we turn to the likely poor's choice of λ in period 0. Due to the discontinuity of the policy outcome, the likely poor really have only two options to choose from. They either form optimal beliefs among those which support high taxation or optimal beliefs among those which support low taxation. We now derive the conditions under which the likely poor choose optimism and low taxation over realism and high taxation. In other words, we derive the conditions under which the prospects of upward mobility of the likely poor are so high, that a low tax regime is supported.

Let $\overline{\lambda}$ be the optimal recall rate given $\lambda \geq \frac{1}{2(1-q)}$ and $\underline{\lambda}$ the optimal recall rate given $\lambda < \frac{1}{2(1-q)}$. If the likely poor choose $\overline{\lambda}$, the tax rate will be $\tau^* = 1$. The expected utility then is

$$U_{0,i}^{\overline{\lambda}} = \overline{\lambda} u_{0,i}(y_L, y_L, 1) + (1 - \overline{\lambda}) u_{0,i}(y_L, y_H, 1) = \delta s \bar{y} + \delta^2 \bar{y}. \tag{12}$$

Whether they end up being optimists or realists does not matter since in both cases they

[14]Or rather we let agents form their beliefs strategically taking account how it affects the policy outcome. Technically speaking the voting here is sincere but agents can affect their policy preferences via their beliefs. The assumption that the policy outcome is $\overline{\tau}$ in case of $\lambda = \frac{1}{2(1-q)}$ ensures that an optimal choice of λ exists for all $s > 0$.

[15]In contrast to models of Bénabou and Tirole (2006) and Bénabou (2008), where voting is sincere, here the possibility of losing income due to less redistribution is the only thing that restricts the optimism of voters. This lets us focus on the trade-off between anticipation and redistribution. Sincere voting is studied in section 4.3.

expect the redistribution to equalize all incomes. If they, on the other hand, choose $\underline{\lambda}$, the tax rate will be $\tau^* = 0$. The expected utility is

$$\begin{aligned} U_{0,i}^{\underline{\lambda}} &= \underline{\lambda} u_{0,i}(y_L, y_L, 0) + (1-\underline{\lambda}) u_{0,i}(y_L, y_H, 0) \\ &= \underline{\lambda}[\delta s y_L + \delta^2 y_L] + (1-\underline{\lambda}) \left[\delta s[r(\underline{\lambda}) y_H + (1-r(\underline{\lambda})) y_L] + \delta^2 y_L\right]. \end{aligned} \tag{13}$$

With probability $\underline{\lambda}$, a likely poor agent recalls $\hat{\sigma}_i = y_L$ and forms realistic beliefs, and with probability $1-\underline{\lambda}$, a likely poor agent recalls $\hat{\sigma}_i = y_H$ and forms optimistic beliefs weighted by the reliability of the signal. In both cases she still ends up consuming y_L in period 2.

The comparison of (12) and (13) tells us if the likely poor would rather choose high anticipatory utility in period 1 and low taxation with low consumption in period 2 over low anticipatory utility and high taxation with high consumption. The difference between the utilities resulting from these two choices, which we call the *incentive to optimism*, can be written as:

$$U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} = -\delta^2(\bar{y} - y_L) + s\delta[\underline{\lambda} y_L + (1-\underline{\lambda})[r(\underline{\lambda}) y_H + (1-r(\underline{\lambda})) y_L] - \bar{y}]. \tag{14}$$

The first term tells what a likely poor agent loses in income and consumption if the tax rate is $\tau^* = 0$ instead of $\tau^* = 1$. The second term tells what she expects to gain in anticipatory utility if she chooses $\underline{\lambda}$ instead of $\overline{\lambda}$. The likely poor are better off in the low tax regime if the incentive to optimism is positive. That is, if $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} > 0$ the likely poor agents choose $\lambda = \underline{\lambda}$.

Lemma 1 (Awareness choices of the likely poor, $\tau \in [0,1]$). *When $\tau \in [0,1]$, the likely poor choose $\lambda = \underline{\lambda} = 0$ if*

$$s > s^*(\chi) \equiv \delta \frac{q + \chi(1-q)}{(1-\chi)(1-q)}. \tag{15}$$

Otherwise they choose $\lambda = \overline{\lambda} \in [\frac{1}{2(1-q)}, 1]$.

We have defined s^* to be a threshold such that if $s > s^*$, then agents value anticipation enough for the gain in anticipatory utility to outweigh the loss of income, and the likely poor will be optimistic enough to vote for a low tax rate. If, on the other hand, $s < s^*$, then the anticipation is not enough to compensate for the lost income and the likely poor will remain realistic enough to vote for a high tax rate.

Lemma 2 (**Politico-economic equilibria,** $\tau \in [0,1]$)**.** *A politico-economic equilibrium is a 4-tuple* $(y_H, \lambda^*, r(\lambda^*|\chi), \tau^*)$.[16]

(i) If $s > s^*$*, there is an equilibrium in which the likely poor choose* $\lambda^* = \underline{\lambda} = 0$*, the likely rich choose* $\hat{\sigma} = y_H$*, and the policy outcome is* $\tau^* = 0$.

(ii) If $s < s^*$*, there are equilibria in which the likely poor choose* $\lambda^* = \overline{\lambda} \in [\frac{1}{2(1-q)}, 1]$*, the likely rich choose* $\hat{\sigma} = y_H$*, and the policy outcome is* $\tau^* = 1$.

The POUM effect occurs in the equilibrium (i), so the condition for the possibility of the POUM effect is equivalent to the condition of the equilibrium (i).

Proposition 1 (**The condition for the POUM effect,** $\tau \in [0,1]$)**.** *When* $\tau \in [0,1]$*, the condition for the POUM effect is* $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} > 0 \iff s > s^*$.

The prospects of upward mobility lead to low taxes if agents value anticipatory utility enough. How much is enough depends on the threshold s^*. The higher s^* is, the less likely the POUM effect is, and conversely, the lower s^* is, the more likely we will observe low taxation. This threshold varies with the parameters of the model. First, the POUM effect becomes more likely with discounting. Myopic preferences put more weight on anticipation which occurs before consumption.[17] Second, the effects of changes in the income distribution are left for section 4.1. Third, the threshold depends on the degree of Bayesian sophistication χ, which we study more closely now.

Consider first the special case of completely naive inference. Setting $\chi = 0$, we get

$$s^*(0) = \delta \frac{q}{1-q}. \tag{16}$$

This special case corresponds to Minozzi's (2013) model.[18] If, on the other hand, we let agents' inference approach Bayesian rationality, we find:

$$\lim_{\chi \to 1} s^*(\chi) = \infty. \tag{17}$$

[16] There is actually a third type of equilibrium, where all agents choose $\hat{\sigma}_i = y_H$ and the policy outcome is $\tau^* = 0$ even if $s < s^*$. There would be no unilateral incentive to deviate. This equilibrium would be the unique equilibrium if we assumed sincere voting.

[17] Interestingly, in the model of Bénabou and Ok (2001), discounting makes the POUM effect less likely. This result in their model is, however, derived in a multiperiod setting and is not directly comparable.

[18] Minozzi's model which abstracts from discounting derives $\delta^* = \frac{n-m}{m}$, where δ^* is the threshold of the savoring parameter, n is the (finite) number of agents, and m is the number of the likely poor.

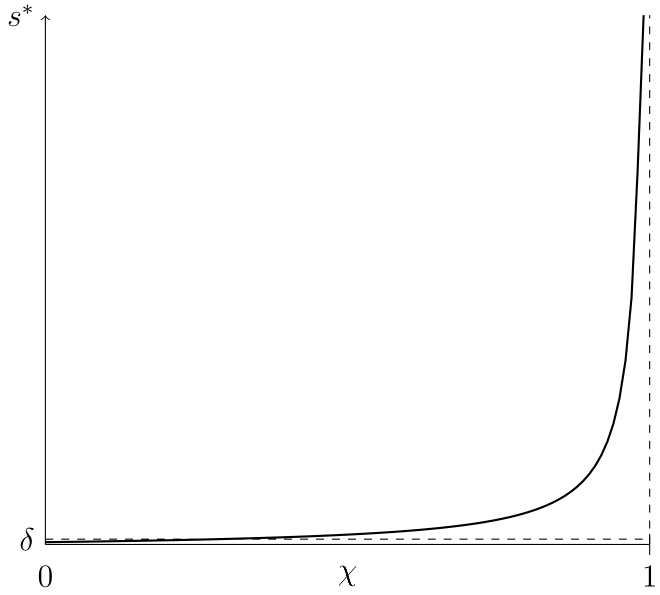

Figure 2: s^* as a function of χ

The threshold required for the POUM effect to occur approaches infinity as the inference of agents approaches full Bayesian rationality. This means that with full Bayesian rationality the importance of anticipation s can never be above s^* and it can never be optimal for the likely poor to form beliefs that support low taxes as the policy outcome. That is, on contrary to the special case of Minozzi's (2013) model, where $\chi = 0$, if we acknowledge that the people cannot simply choose their beliefs and let $\chi > 0$, the threshold s^* increases dramatically in χ and in the extreme case of full Bayesian rationality, the POUM effect can never occur.

Figure 2 tracks the threshold s^* as a function of χ. To give some concreteness to the results here, we note from the period 0 utility in (9) that if $s = \delta$, then agents value anticipatory utility as much as consumption. The dashed line in Figure 2, denoted by δ, depicts this value of s. For the threshold values $s^* > \delta$, the anticipation of consumption must bring more utility to the agents than the consumption itself to make the POUM effect possible. We see that s^* is below δ only for very small values of χ.

To see why fully Bayesian likely poor agents can never be better off with low taxes, consider again the incentive to optimism given in (14). Plugging in the optimal recall rate $\underline{\lambda} = 0$, the incentive to optimism can be written as

$$U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} = -\delta^2(\bar{y} - y_L) + s\delta[r(0|\chi) - q]\Delta y, \tag{18}$$

where $\Delta y \equiv y_H - y_L$. The second term in the right hand side is the gain in anticipation if an agent chooses $\underline{\lambda}$ over $\overline{\lambda}$. Noting that $r(0|\chi) \to 1$ as $\chi \to 0$ and $r(0|\chi) \to q$ as $\chi \to 1$, it is easy to see how the value of the second term goes to zero as $\chi \to 1$ and why it does not when $\chi = 0$. The incentive to optimism is at its maximum when $\chi = 0$ and as agents' inference approaches full Bayesian rationality the utility gain from anticipation vanishes.

The reliability which the agents use to weight the information of their recollection plays a crucial role here. For $\chi = 1$, the reliability $r(\lambda|\chi)$ is an increasing function of λ. The more realistic the likely poor are, the more reliable signal $\hat{\sigma}_i = y_H$ is. On the other hand, when the likely poor systematically memorize and recall $\hat{\sigma}_i = y_H$, they know that no matter what is their true signal, they recall $\hat{\sigma}_i = y_H$. In this case, the signal does not carry any information anymore, and agents form their beliefs relying on the prior distribution, $r(0|\chi) = q$. However, when the degree of Bayesian sophistication decreases, the reliability becomes less and less dependent on λ, and the optimistic poor put more and more weight on their pleasant recollection. When $\chi = 0$, the reliability is independent of λ and no matter how optimistic the likely poor are, they always fully trust their recollections.

It is instructive to see how the period-0 expectation of expected period-2 income in period 1, and expected anticipatory utility which is proportional to the expected income, varies with λ and χ. For this, we shortly abstract from taxation to see how the choice of λ and the sophistication of agents' inference interact in forming the belief about their future gross income. The expectation of expected gross income of a likely poor agent in period 1 from the point of view of period 0 as a function of λ is[19]

$$\iota_{gross}(\lambda|\chi) \equiv E[E[y_i|F_{1,i}]|\lambda, \chi, F_{0,i}] = (1-\lambda)[r(\lambda)y_H + (1 - r(\lambda))y_L] + \lambda y_L. \tag{19}$$

This function is plotted in Figure 3 for different values of χ. The lowest curve corresponds to the case $\chi = 1$. As agents put more and more weight on signal $\hat{\sigma}_i = y_H$ in their period 0 strategy, that is, as they become more and more likely to remember $\hat{\sigma}_i = y_H$, the expected income approaches the average income. In the case of $\lambda = 0$, each of the likely poor and each of the likely rich always recall signal $\hat{\sigma}_i = y_H$. As everyone is pooling on the same signal, receiving this signal does not give any information, and agents rely on the prior information when assessing their future income. In the case of full Bayesian rationality, it is therefore not possible for agents to achieve above average expectations.

[19]See the discussion in section 3.2

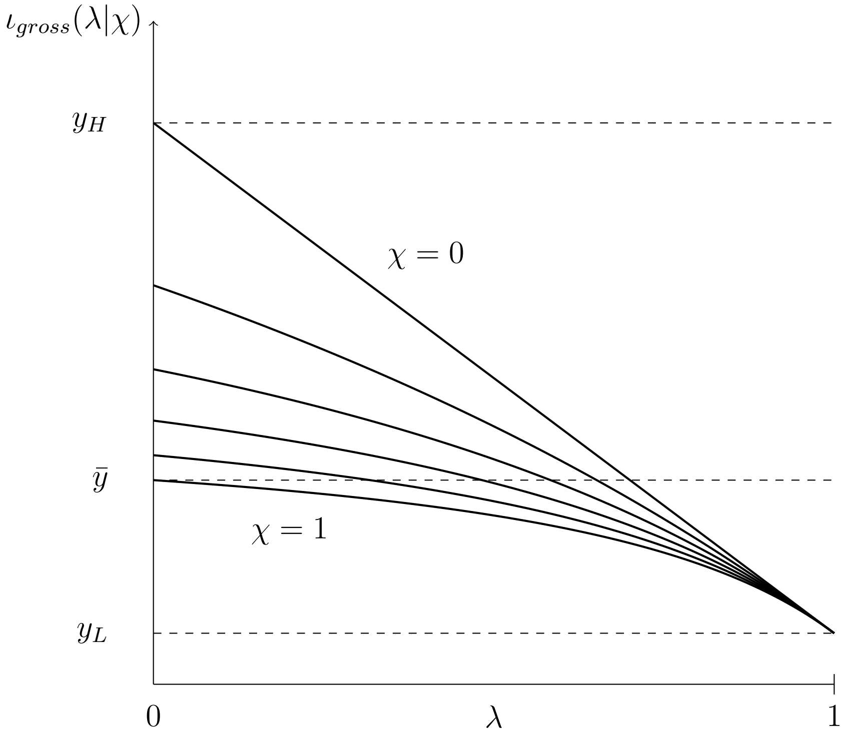

Figure 3: $\iota_{gross}(\lambda|\chi)$ for different values of χ

As they expect average income in the fully expropriating high tax regime, they cannot possibly improve their utility by voting for low taxes.

On the contrary, when $\chi < 1$, agents can achieve above average expectations, and they, therefore, can have a gain in anticipatory utility to trade off against the lost income in the low tax regime. For agents with $\chi < 1$, a decrease in λ does not affect the reliability of the signal as much as it affects for the Bayesian rational agents. In the limiting case of $\chi = 0$, represented by the linear curve in Figure 3, the reliability is independent of λ, and all agents can believe to be of type y_H. The expectations of naive agents are not as constrained as the expectations of Bayesian agents and the more naive the agents are, the less constrained their beliefs are. The naive agents can, therefore, achieve higher hopes and higher anticipatory utility than their Bayesian counterparts.

What values of χ are feasible then? Do people have the introspection to realize that they might have a self-serving tendency to remember positive news and forget bad news or are they always able to deceive themselves into believing what fits them best? Minozzi (2013) justifies his assumption of full naivete by arguing that the belief formation is an automatic and unconscious process and therefore the agents cannot recall the process itself and are therefore ignorant of it occurring. They then completely trust their recollections, since they have forgotten the action of their past self or rather since they never even knew about the action of their unconscious self. On the other hand, Bénabou and Tirole (2002) argues that if a person consistently memorizes good news and ignores bad news, she will likely become aware of this tendency and will therefore not fully rely on her recollections. So even if the belief formation is an automatic, unconscious process and people cannot, therefore, recall it happening, they, by learning from their past mistakes, will internalize the existence of this process and start adjusting their reliance on their memories accordingly.

Framed in other words, the implausible consequence of assuming $\chi = 0$ is that people are able to choose their beliefs without them in any way depending on the objective reality. To be clear, the beliefs supplied by a naive cognitive technology are usually restricted to the support of the outcome and can be further constrained to a subset of the support.[20] Also, a naive cognitive technology does take the reality into account, when the beliefs are traded against their adverse consequences. However, in principle, it does not need to. Naive cognitive technologies are also nevertheless insensitive to the distribution of outcomes. When $\chi = 0$, an agent can believe to be likely rich no matter how small the prior probability of being rich is given that this prior probability is positive. Even if the belief formation mechanism is an automatic and unconscious process, it seems implausible that this process does not need in any way to take account the information that the reality inevitably provides, and that people can simply choose their beliefs. Indeed, Kunda (1990) strongly argues that people do not seem to be completely free to believe what they want to believe. According to him, people can bias their beliefs only to the extent that they can justify their new beliefs. The main mechanism for the justification of the new beliefs is a biased memory search which implies that prior beliefs do play a role in determining the new beliefs. Also, according to evidence, changes in beliefs seem to be constrained by pre-existing beliefs (Kunda, 1990). Therefore, a belief formation technology with some Bayesian sophistication, which anchors the beliefs to the prior distribution, and therefore to the reality in our model, would seem more plausible a representation of these psychological processes than a belief formation technology with none Bayesian sophistication.

However, assuming $\chi = 1$ is rather extreme as well and $\chi \in (0, 1)$ would most likely best reflect the reasoning of real people. Bénabou and Tirole (2002) presents the model

[20]See the footnote 2 in the appendix to Minozzi (2013).

in the context of people distorting their beliefs to motivate themselves when facing time inconsistency problems. This might be a context in which people get enough feedback to learn about their unconscious information processing. In the context of the present work, where people form beliefs about their future incomes and vote for redistribution, the feedback mechanism may not facilitate this learning. The actions taken are long-lasting, there are not that many chances of learning, and the real-life mechanism with which votes transform to redistributive policies is noisy and complicated. It might, therefore, be plausible that the sophistication in the belief formation process depends on the context and that, indeed, in the context of forming beliefs about future income, people might be less sophisticated as in the context of motivating oneself in the everyday activities.

In the case of full Bayesian rationality, the wishful beliefs of the likely poor are bounded above to the average income, which is what they expect to receive if $\tau^* = 1$ as well. They cannot, therefore, increase their anticipatory utility by distorting their beliefs. However, what if they could not expect the incomes to be fully equalized under the high tax policy. Then they might be able to increase their anticipatory utility by distorting their beliefs even if they still ended up with expectations of average income. The case of $\overline{\tau} = 1$ and $\underline{\tau} = 0$ is maybe a bit too unrealistic a simplification and we therefore turn now to the general case of $\tau \in [\underline{\tau}, \overline{\tau}]$.

3.6 Conditions for the POUM effect, $\tau \in [\underline{\tau}, \overline{\tau}]$

The weakness of the previous setting is that if agents vote for full expropriation, they know that their period 2 incomes will be the average income. Therefore, no matter what they believe, they will expect average income. On the other hand, if the likely poor choose optimism, they will lose all redistribution, which is a very high cost for optimism. To address these problems, we now consider the general case of our model and impose lower and upper limits on the tax rate. That is, we now set $\tau \in [\underline{\tau}, \overline{\tau}]$, and require $\underline{\tau} < \overline{\tau}$ so that the set of tax policies is always nonempty.

If we set $\overline{\tau} < 1$, the anticipated consumption and the consumption of the likely poor realists will now be below average in the high tax regime. This makes the high tax equilibrium less attractive compared to the case of $\overline{\tau} = 1$. The increase in payoff when choosing optimism over realism is therefore now greater, and the condition for the POUM effect should become looser.

At the other extreme, a full laissez-faire policy is not a completely innocuous simplification either. The likely poor have to trade optimism against losing all redistribution. Imposing a lower limit for redistribution makes this trade-off less drastic. If $\underline{\tau} > 0$, there will be some taxation in the low tax regime as well, and the consequences of optimism are less severe for the likely poor. By setting $\underline{\tau} > 0$, we make the decrease in period 2 consumption of the likely poor smaller in case they choose $\underline{\lambda}$ over $\overline{\lambda}$. Again, this should make optimism more attractive and the POUM effect more likely. Of course, an increase in $\underline{\tau}$ decreases the anticipatory utility of the optimists, but the effect in period 2 consumption seems to dominate for the likely poor.

To put these effects together, by restricting the set of available tax policies, we make the POUM effect more feasible in two ways. First, by decreasing the attractiveness of realism by having lower taxes and, therefore, lower anticipation and consumption in the high tax regime. Second, by increasing the attractiveness of optimism by having higher taxes in the low tax regime and therefore higher consumption but possibly lower anticipation. The effects on the period 2 consumption and realists' anticipation seem to dominate the effect on optimists' anticipation so that the smaller is the range of allowed tax policies, the more likely the POUM effect occurs.

As before, the apparently continuous choice reduces to a binary choice, and the likely poor choose between $\overline{\lambda}$ and $\underline{\lambda}$ knowing that choosing the former leads to high taxation and choosing the latter leads to low taxation. If they choose the former, the tax rate will be $\tau^* = \overline{\tau}$ and their expected payoffs are

$$
\begin{aligned}
U_{0,i}^{\overline{\lambda}} &= \overline{\lambda} u_{0,i}(y_L, y_L, \overline{\tau}) + (1 - \overline{\lambda}) u_{0,i}(y_L, y_H, \overline{\tau}) \\
&= \overline{\lambda}\big[\delta s[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}]\big] \\
&+ (1-\overline{\lambda})\big[\delta s[(1-\overline{\tau})[r(\overline{\lambda})y_H + (1 - r(\overline{\lambda}))y_L] + \overline{\tau}\bar{y}] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}]\big]. \qquad (20)
\end{aligned}
$$

This differs from (12) in that the tax does not fully equalize the incomes. When all incomes are not equalized, different expectations lead to different amounts of anticipation. This allows the anticipatory utility of optimists and realists to diverge also in the high tax regime. Note especially how a realist derives anticipatory utility from an expectation of below average income.

If the likely poor agents choose the latter, the tax rate will be $\tau^* = \underline{\tau}$ and their expected payoffs are

$$\begin{aligned}
U_{0,i}^{\underline{\lambda}} &= \underline{\lambda} u_{0,i}(y_L, y_L, \underline{\tau}) + (1-\underline{\lambda}) u_{0,i}(y_L, y_H, \underline{\tau}) \\
&= \underline{\lambda}\big[\delta s[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}]\big] \\
&+ (1-\underline{\lambda})\big[\delta s[(1-\underline{\tau})[r(\underline{\lambda})y_H + (1-r(\underline{\lambda}))y_L] + \underline{\tau}\bar{y}] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}]\big].
\end{aligned} \tag{21}$$

With probability $\underline{\lambda}$, a likely poor agent ends up being a realist and anticipates low consumption. With probability $1-\underline{\lambda}$, she ends up being an optimist and anticipates high consumption. In both cases the period 2 consumption is low. However, in comparison to (13), the period 2 consumption is now higher, and the consequences of optimism are now less severe for the likely poor.

The incentive to optimism is

$$\begin{aligned}
U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} &= -\delta^2(\overline{\tau} - \underline{\tau})(\bar{y} - y_L) + \delta s\big[(1-\underline{\lambda})(1-\underline{\tau})r(\underline{\lambda})\Delta y + (1-\underline{\tau})y_L + \underline{\tau}\bar{y}\big] \\
&- \delta s\big[(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})\Delta y + (1-\overline{\tau})y_L + \overline{\tau}\bar{y}\big].
\end{aligned} \tag{22}$$

To not clutter the page with notation, the incentive to optimism is written in a still interpretable, but different and a more compact form than (14). The first term tells the loss of income due to less redistribution. The second term measures the expected anticipatory utility if the the likely poor choose $\lambda = \underline{\lambda}$. With probability $1 - \underline{\lambda}$ there is an increase of $(1-\underline{\tau})r(\underline{\lambda})\Delta y$ from the "base level" of $(1-\underline{\tau})y_L + \underline{\tau}\bar{y}$ in anticipatory utility. The third term similarly measures the expected anticipatory utility if the likely poor choose $\lambda = \overline{\lambda}$. If the incentive to optimism is positive, the likely poor will prefer to be optimists and choose $\lambda = \underline{\lambda}$.

Lemma 3 (Awareness choices of the likely poor, $\tau \in [\underline{\tau}, \overline{\tau}]$). *When $\tau \in [\underline{\tau}, \overline{\tau}]$, the likely poor choose $\lambda = \underline{\lambda} = 0$ if*

$$s > s^{**}(\chi) \equiv \frac{\delta(\overline{\tau} - \underline{\tau})q}{(1-\underline{\lambda})(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q}. \tag{23}$$

Otherwise they choose $\lambda = \overline{\lambda} = \frac{1}{2(1-q)}$.

As before, whether the savoring parameter is above or below the threshold s^{**}, the likely poor will either prefer high anticipation with low redistribution or low anticipation with high redistribution. The choice of the likely poor determines the tax rate.

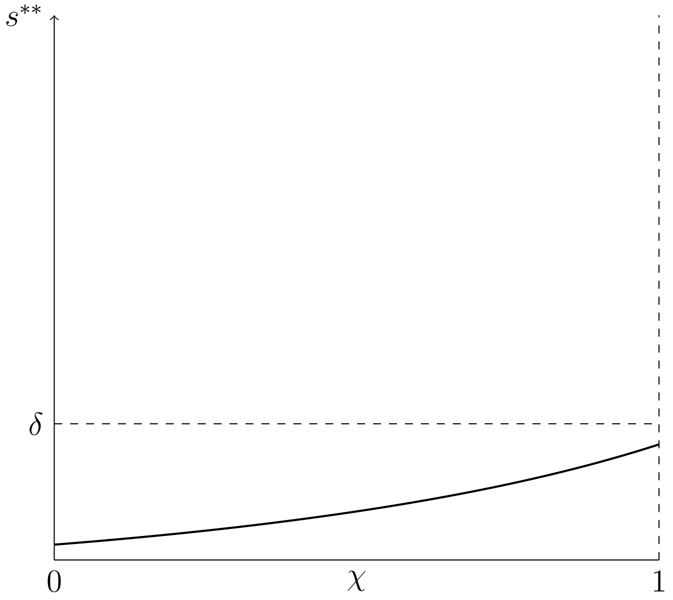

Figure 4: s^{**} as a function of χ

Lemma 4 (**Politico-economic equilibria,** $\tau \in [\underline{\tau}, \overline{\tau}]$)**.** *A politico-economic equilibrium is a 4-tuple* $(y_H, \lambda^*, r(\lambda^*|\chi), \tau^*)$.[21]

(i) If $s > s^{**}$, *there is an equilibrium in which the likely poor choose* $\lambda^* = \underline{\lambda} = 0$, *the likely rich choose* $\hat{\sigma} = y_H$, *and the policy outcome is* $\tau^* = \underline{\tau}$.

(ii) If $s < s^{**}$, *there is an equilibrium in which the likely poor choose* $\lambda^* = \overline{\lambda} = \frac{1}{2(1-q)}$, *the likely rich choose* $\hat{\sigma} = y_H$, *and the policy outcome is* $\tau^* = \overline{\tau}$.

As before, the POUM effect occurs in the equilibrium (i) and the conditions for the POUM effect are the same as the conditions for this equilibrium.

Proposition 2 (**The condition for the POUM effect,** $\tau \in [\underline{\tau}, \overline{\tau}]$)**.** *The condition for the POUM effect is* $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} > 0 \iff s > s^{**}$.

Interestingly, s^{**} is now finite for all $\chi \in [0, 1]$. In contrast to the setting in the previous section, the POUM effect becomes possible even if the agents are fully Bayesian information processors. Figure 4 depicts s^{**} as a function of χ. We see that the threshold s^{**} does not increase in χ as sharply as s^* does. As before, to ease the interpretation, the dashed line depicts the values of s for which the agents derive as much utility from the anticipation of consumption as from consumption itself. The parameter values for

[21] There is actually a third type of equilibrium, where all agents choose $\hat{\sigma} = y_H$ and the policy outcome is $\tau = 0$ even if $s < s^{**}$ as there would be no unilateral incentive to deviate.

the allowed tax policies used in Figure 4 are $\underline{\tau} = 0.25$ and $\overline{\tau} = 0.45$, and they represent roughly the total tax revenues as a percentage of the gross domestic product in the US and in the Nordic Countries, respectively (OECD, 2018).[22] These values and countries are chosen to represent the extremes of taxation among the developed countries and serve only as an example. The hypothetical extremes of tax policies are probably larger than currently existing extremes. As we will see, the bounds of allowed tax policies have a clear effect on s^{**}.

The following proposition makes formal the effect of the naivete parameter χ which can be seen in Figure 4.

Proposition 3 (Effect of change in the degree of Bayesian sophistication). *The partial derivative of s^{**} with respect to χ is positive, that is, $\frac{\partial s^{**}}{\partial \chi} > 0$ for all parameter values. The more sophisticated the cognitive technology is, the less likely is the POUM effect.*

Even if the POUM effect is now possible for all $\chi \in [0, 1]$, it can still be questioned whether it is feasible for all $\chi \in [0, 1]$. Again, the agents may have to value anticipation more than consumption to prefer low taxes if the range of the feasible tax rates is big enough. To see this, consider the threshold value s^{**} when $\chi = 1$:

$$s^{**}(1) = \delta \frac{(\overline{\tau} - \underline{\tau})(1 - q)}{(1 - \overline{\tau})q}. \tag{24}$$

Now $s^{**} > \delta$, for all pairs $(\underline{\tau}, \overline{\tau})$, such that $\overline{\tau} > (1 - q)\underline{\tau} + q$. We could argue that within a jurisdiction, the range of feasible tax rates is small enough and hence, the POUM effect is feasible also for a sophisticated cognitive technology. On the other hand, as discussed, fully Bayesian sophistication may not be the correct specification in the belief distortion technology to represent people's beliefs about their future incomes and their voting behavior. Certainly, the set of values of χ for which the POUM effect is feasible has now increased in comparison to the case in the previous section.

To understand how the likelihood of the POUM effect depends on the maximum and minimum taxes, consider first what happens when we set an upper limit on the tax rate. The upper limit of the tax is relevant when the likely poor choose $\lambda = \overline{\lambda}$, since then the resulting policy is high taxes. By imposing a restriction on how much of the income can be redistributed we make the prospects of choosing $\lambda = \overline{\lambda}$ worse. Consider the effects on

[22] $q = 0.3$ and δ is normalized to 1. Note that the curve is independent of the values of y_L and y_H.

the period 2 consumption and period 1 anticipatory utility separately. First, a decrease in the upper limit of the tax rate decreases the period 2 consumption of the likely poor in the high tax regime, which makes voting for high taxes less rewarding. Second, for those of the likely poor who end up being realists, the lower consumption in period 2 implies lower anticipation in period 1. Those of the likely poor who end up being optimists will expect above-average incomes, and they will, therefore, gain in anticipatory utility as the upper limit of the tax decreases. However, it can be shown that this latter effect is dominated and the effect on ex-ante expected anticipation stays negative.[23] That is, when imposing an upper limit for the tax rate, both anticipation and consumption prospects of choosing $\lambda = \overline{\lambda}$, that is, of being realist, deteriorate. Proposition 4 formalizes this total effect of the upper limit of the tax rate.

Proposition 4 (Effect of upper limit of tax rate on the conditions for POUM). *The partial derivative of s^{**} with respect to $\overline{\tau}$ is positive, that is, $\frac{\partial s^{**}}{\partial \overline{\tau}} > 0$ for all parameter values. The POUM effect becomes more likely as $\overline{\tau}$ decreases.*

Consider next what happens when we set a lower limit for the allowed tax rate. The prospects of choosing $\lambda = \underline{\lambda}$, on the other hand, are now better. The likely poor choosing $\lambda = \underline{\lambda}$ leads to low taxes, so here the lower limit of the tax rate is interesting. Again, there is an effect on the period 2 consumption and on the period 1 anticipation. First, even if the likely poor vote for low taxation, redistribution does not vanish altogether. Since they are trading their optimism against redistribution, the cost of optimism is now lower. The reduction in their period 2 consumption is not as big as with the possibility of complete laissez-laire. This makes choosing high anticipation and low taxes more attractive. Second, when choosing $\lambda = \underline{\lambda}$, all of the likely poor end up being optimists. If they then anticipate above average income, that is, if $\chi < 1$, then an increase in the lower limit of the tax rate will decrease their anticipatory utility. The less sophisticated the agents are, the more they expect to earn, and the higher is the decrease in their anticipation. The effect on anticipatory utility is opposite to the effect on consumption. The effect on consumption, however, seems to dominate. Proposition 5 formalizes this.

Proposition 5 (Effect of lower limit of tax rate on the conditions for POUM). *The partial derivative of s^{**} with respect to $\underline{\tau}$ is negative, that is, $\frac{\partial s^{**}}{\partial \underline{\tau}} < 0$ for all parameter values. The POUM effect becomes more likely as $\underline{\tau}$ increases.*

[23] $\frac{\partial}{\partial \overline{\tau}} \iota_{net}(\overline{\lambda}, \overline{\tau}) = [q - (1 - \overline{\lambda}) r(\overline{\lambda})] \Delta y > 0$, where $\iota_{net}(\cdot)$ is defined below.

To summarize these effects, the utility from choosing $\lambda = \underline{\lambda}$ increases with the lower bound of the tax rate and the utility from choosing $\lambda = \overline{\lambda}$ decreases when we impose an upper bound for the tax rate. This means that the utility cap between choosing $\lambda = \underline{\lambda}$ and $\lambda = \overline{\lambda}$ increases as the range of allowed tax policies decreases. This utility cap is, by definition, the incentive to optimism. An increase in the incentive to optimism then leads to less stringent conditions for the POUM effect.

To gain further intuition on the conditions for the POUM effect, write s^{**} as

$$s^{**} = \frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)}{\iota_{net}(\underline{\lambda}, \underline{\tau}) - \iota_{net}(\overline{\lambda}, \overline{\tau})} \tag{25}$$

where

$$\iota_{net}(\lambda, \tau) \equiv \lambda[(1-\tau)y_L + \tau\bar{y}] + (1-\lambda)[(1-\tau)(r(\lambda)y_H + (1-r(\lambda))y_L) + \tau\bar{y}] \tag{26}$$

is the ex ante expectation of the expected consumption of the likely poor in period 1 given the choice of λ and the resulting tax policy τ, and where $\underline{\lambda} = 0$, and $\overline{\lambda} = \frac{1}{2(1-q)}$.[24] The nominator of (25) represents the difference in period 2 consumptions in the two different tax regimes. Clearly, when $\overline{\tau}$ decreases or $\underline{\tau}$ increases, this difference becomes smaller. As discussed, when this difference becomes smaller the loss in the period 2 consumption when choosing $\lambda = \underline{\lambda}$ over $\lambda = \overline{\lambda}$ decreases. If the nominator decreases, s^{**} decreases proportionally and the POUM effect becomes more likely. The denominator of (25) is proportional to the difference in expected anticipatory utility of the likely poor between their choices of low or high recall rate. When this difference increases, the likely poor have more to gain in anticipation and belief distortion becomes more attractive. If the denominator increases, s^{**} decreases and the POUM effect becomes more likely.

In choosing their awareness rate, the likely poor agents make a trade-off between anticipatory utility and consumption. By imposing limits on possible tax rates, we alter this trade-off such that they have less to lose in consumption. The stakes of wrong decisions due to biased beliefs are now smaller, and optimism is, therefore, more attractive.

[24] From this expression it is simple to derive Minozzi's (2013) result in another form. By setting $\underline{\tau} = 0$, $\overline{\tau} = 1$, $\delta = 1$, and $\chi = 0$, we get $s^{**} = \frac{\bar{y} - y_L}{y_H - \bar{y}}$. Minozzi's (2013) condition for the POUM effect is $\delta > \delta^* = \frac{\bar{y} - y_p}{y_r - \bar{y}}$, where y_p is the income of the likely poor, y_r income of the likely rich, and δ^* the threshold in the savoring parameter δ.

4 Further Analysis

4.1 Effects of changes in the income distribution

As already seen, given the value that agents put on anticipation s, the threshold s^{**} determines whether the POUM effect occurs. The comparative statistics of s^{**}, therefore, reveal how the conditions for the POUM effect vary as the parameters of the model change. In this section we consider the effects of changes in y_L, y_H, $\bar{y}$, and q.

Following Minozzi's (2013) analysis, we first examine the changes in y_L and y_H holding the average income constant. Proposition 6 collects these results.

Proposition 6 (Effects of changes in y_L and y_H holding $\bar{y}$ constant). *Holding the average income constant, the threshold s^{**} decreases in y_L and y_H, that is, the POUM effect becomes more feasible when y_L or y_H increase.*[25]

If the incomes of the likely rich increase such that the average income stays constant, the conditions for the POUM effect become looser. Similarly, if the incomes of the likely poor increase such that the average income stays constant, the conditions for the POUM effect become again looser. We insist on holding the average income constant because it makes the effects interesting. The average income $\bar{y}$ is a function of both y_L and y_H and taking this into account gives us $\frac{\partial s^{**}}{\partial y_H} = \frac{\partial s^{**}}{\partial y_L} = 0$ as can easily be seen by noting that s^{**} in (23) is independent of both y_H and y_L. So by letting the average income adapt to the changes in the incomes of the likely poor or the likely rich, the condition for the POUM effect would not change.

Holding the average income constant might feel artificial, but looking at the incentive to optimism given in (22) gives us an idea, what the partial derivatives holding the average income constant mean here.[26] For the agents, changes in the average income imply changes in the transfers they receive, whereas changes in either y_L or y_H imply changes in the expectations of their pre-tax income. That is, holding average income constant means holding the tax revenue and transfers constant, whereas increases in the high and low levels of income mean increased expectations of gross income. Increased prospects of gross income, when the transfers are expected to stagnate, make optimism more rewarding. This kind of change in the income distribution could occur, for instance, if the income tax is regressive such that the increase in the incomes of the likely rich

[25] These effects are the same for s^*

[26] Unfortunately, Minozzi (2013) does not justify this choice in his comparative analysis.

does not lead to a proportional increase in the tax revenue. We could also interpret the income levels y_L and y_H more loosely as what the likely poor perceive these income levels to be. The perceived income of the likely rich could change without affecting the tax revenue, for instance, if the incomes in other jurisdictions change and the likely poor observe this or if the consumption habits of the likely rich change towards more conspicuous consumption. *"In 1972, a storm of protests from blue-collar workers greeted Senator McGovern's proposal for confiscatory estate taxes. They apparently wanted some big prizes maintained in the game. The silent majority did not want the yacht clubs closed forever to their children and grandchildren while those who had already become members kept sailing along."* writes Okun (2015, p. 47).

Similarly, the change in the average income has no effect as such, $\frac{\partial s^{**}}{\partial \bar{y}} = 0$, but holding y_L and y_H constant and letting $\bar{y}$ change gives us

Proposition 7 (Effect of change in $\bar{y}$ holding y_L and y_H constant). *Holding y_L and y_H constant, the threshold s^{**} increases in $\bar{y}$, that is, the POUM effect becomes less feasible when the average income increases.*

The case of holding y_L and y_H constant and letting $\bar{y}$ change mirrors the previous discussion. If the likely poor expect increased transfers but the prospects of gross income stay the same, then realism becomes more attractive.

The changes in the fraction of the likely rich produce slightly more complicated effects mainly because the reliability is a function of q, and the optimal recall rate $\overline{\lambda}$ varies with q. We therefore only characterize the effects. Consider a change in the income distribution where the proportion of the likely rich becomes smaller. A decrease in q has three effects. First, it decreases the average income and the tax revenue and, therefore, makes realism less attractive. Second, it decreases $\overline{\lambda}$, the optimal choice if the likely poor opt for high taxes. When there are fewer likely rich agents voting for low taxes, it allows the likely poor to be more optimistic even if they opt for high redistribution. This makes realism more attractive. Third, as q and, hence, $\overline{\lambda}$ decrease, they both contribute to decreasing the reliability of the signal $\hat{\sigma}_i = y_H$ and, therefore, make the anticipated income lower and optimism less attractive.

All effects that work via the reliability of recalled signal depend crucially on χ. Hence, for low values of χ, the reliability does not depend that much on the prior distribution or $\overline{\lambda}$ and the first effect dominates. In this case, POUM effect becomes more likely as the prospects of choosing $\lambda = \overline{\lambda}$ are now worse. For high values of χ, the reliability is highly dependent on the prior and $\overline{\lambda}$ and the second and third effect dominate. In this case, a decrease in q makes POUM less likely. For intermediate values of χ, the relative dominance of these effects varies, and the total effect is nonmonotonic.

4.2 Welfare Analysis

In the simple model of the current paper, utilities are linear in period 2 consumption, meaning that the aggregate utility is not sensitive to the distribution of consumption. Therefore, the aggregate utility is trivially maximized by maximizing the anticipation, no matter what the distribution of the consumption ends up being. Hence, the aggregate utility as a measure of welfare is not very informative. This section, therefore, after a brief discussion on the distribution and the aggregation of consumption and anticipation, focuses on the welfare of the likely poor and the likely rich separately.

The utility of each agent in the economy consists of two components: the utility from anticipation and the utility from consumption. Thanks to the additivity of these utilities, we can study the aggregate levels of these two components separately. Furthermore, as the utilities with respect to consumption and anticipation are both linear, we say that the welfare consists of aggregate consumption and aggregate anticipation.

As the redistribution does not produce any wastage, the aggregate consumption stays constant at the average consumption throughout the analysis. Due to the linearity of utility with respect to consumption, the average utility derived from consumption remains constant as well. Only the distribution of the consumption and the utility from consumption between the likely poor and the likely rich varies depending on the chosen tax policy. The higher is the tax rate, the more equally the aggregate consumption is distributed among the likely rich and the likely poor.

The more novel component of welfare is the aggregate anticipation, which is the sum of anticipation of those agents who recalled $\hat{\sigma}_i = y_L$ and of those who recalled $\hat{\sigma}_i = y_H$. A fraction $(1-q)\lambda$ of agents recalls $\hat{\sigma}_i = y_L$ and they anticipate a gross income of y_L. A fraction $q + (1-q)(1-\lambda)$ of agents recalls $\hat{\sigma}_i = y_H$ and they anticipate a gross income of $r(\lambda)y_H + (1-r(\lambda))y_L$. Note especially that those who truly belong to the likely rich anticipate the same gross income as those of the likely poor who recall signal $\hat{\sigma}_i = y_H$. The aggregate anticipatory utility derived from the anticipation of gross income is

$$(1-q)\lambda s y_L + [(1-q)(1-\lambda) + q]s[r(\lambda)y_H + (1-r(\lambda))y_L]. \qquad (27)$$

The aggregate anticipation depends on the constraints of the cognitive technology and the awareness choices of the likely poor. For $\chi = 1$, the aggregate anticipatory utility is constant at $s\bar{y}$. Bayesian rationality imposes a constraint on beliefs such that on average, agents expect average income. Therefore, for the special case of $\chi = 1$, the aggregate anticipation is similar to the aggregate consumption in the sense that only the distribution of the anticipation varies. As the Bayesian constraint is relaxed and values of $\chi < 1$ are allowed, the aggregate anticipation can exceed the anticipation of average income, and it is no more independent of λ. In this case, the aggregate anticipation is maximized at $\lambda = 0$.

The counterintuitive consequence of the assessment of the reliability of recollections is that for all $\chi > 0$, the likely rich will underestimate their future income. If all of the likely poor choose to memorize the signal $\hat{\sigma}_i = y_H$, then all agents, the likely rich and the likely poor, will recall this signal in period 1. When the likely rich are assessing the reliabilities of their recollections, they know that no matter which signal an agent receives in period 0, they will recall $\hat{\sigma}_i = y_H$. In the case of full Bayesian rationality, this means that the signal is uninformative and the likely rich use the prior information to form their expectations and, therefore, underestimate their future income.[27] If, on the other hand, the likely poor choose to memorize the signal they received, then the likely rich, after recalling $\hat{\sigma}_i = y_H$ know that the only way to recall this signal is to be likely rich. In this case, they put a reliability of 1 to their recollection and form accurate expectations.

This dependence of the anticipation of the rich on the awareness choice of the likely poor can be thought of as a negative externality. As λ decreases, the likely poor are more and more optimistic and the likely rich more and more pessimistic. When the likely poor engage in optimism, they redistribute anticipation. If $\chi = 1$, and the likely poor choose $\lambda = 0$, they equalize all anticipation. In this case, the average anticipation is constant, and the gain in anticipatory utility of the likely poor is exactly offset by the loss in the anticipatory utility of the likely rich. The strength of externality and the redistributive effect increases in χ. For completely naive agents, the reliability of

[27]Interestingly, Cruces, Perez-Truglia, and Tetaz (2013) find evidence, that in addition to the poor overestimating their position in the income distribution, the rich tend to underestimate theirs. However, their proposed mechanism is different: Agents estimate the overall income distribution by extrapolating from the incomes of their reference group. If the reference group does not well represent the overall income distribution, the estimates will be biased. Also, underconfidence is a well-documented phenomenon in the literature of psychology and tends to concern those with the best prospects. See, for instance, Moore and Healy (2008).

recollection is independent of λ, and there is no externality.

This externality should, however, not be thought of as a causal relationship between the cognitive processes of different agents, but as an *externality across information states*, as Bénabou and Tirole (2002, p. 907) put it. The likely rich do not underestimate their prospects because the likely poor overestimate theirs, but because they know that had they themselves been likely poor, they might still have memorized the signal $\hat{\sigma}_i = y_H$. The negative externality for the likely rich is, therefore, caused by their own information processing strategy, that is, by their own hypothetical action in an alternative history.

If the likely poor choose the low tax equilibrium with high expectations, they are obviously better off in this equilibrium. The pessimism of the rich, however, raises the rather surprising question of whether the likely rich are worse or better off in the low tax equilibrium. In the standard case, where the agents do not derive utility from anticipation, the rich have higher consumption when paying low taxes and are obviously better off in the low tax equilibrium. When we take the anticipation into the analysis, the rich still have higher period 2 consumption in the low tax equilibrium, but the negative externality due to the optimism of the poor in this equilibrium erodes their anticipation in period 1. We now see, which of these effects dominates.

In the low tax equilibrium, the utility of the likely rich from the viewpoint of period 0 is

$$u_{0,i}(y_H, y_H, \underline{\tau}) = \delta s \left[(1-\underline{\tau})[r(\underline{\lambda})y_H + (1-r(\underline{\lambda})y_L] + \underline{\tau}\bar{y}\right] + \delta^2[(1-\underline{\tau})y_H + \underline{\tau}\bar{y}], \tag{28}$$

and in the high tax equilibrium the utility of the likely rich is

$$u_{0,i}(y_H, y_H, \overline{\tau}) = \delta s \left[(1-\overline{\tau})[r(\overline{\lambda})y_H + (1-r(\overline{\lambda})y_L] + \overline{\tau}\bar{y}\right] + \delta^2[(1-\overline{\tau})y_H + \overline{\tau}\bar{y}]. \tag{29}$$

Again, whether the anticipation effect dominates depends on the importance of anticipation. The likely poor choosing optimism and low taxes makes the likely rich worse off if (29) is greater than (28). If $(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q > 0$, the condition for this reads:

$$s < \frac{-\delta(\overline{\tau}-\underline{\tau})(1-q)}{(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q}. \tag{30}$$

Since the denominator is positive and the nominator negative, the right-hand side of (30) is negative. As $s \geq 0$, the condition is never satisfied, and the likely rich are always better

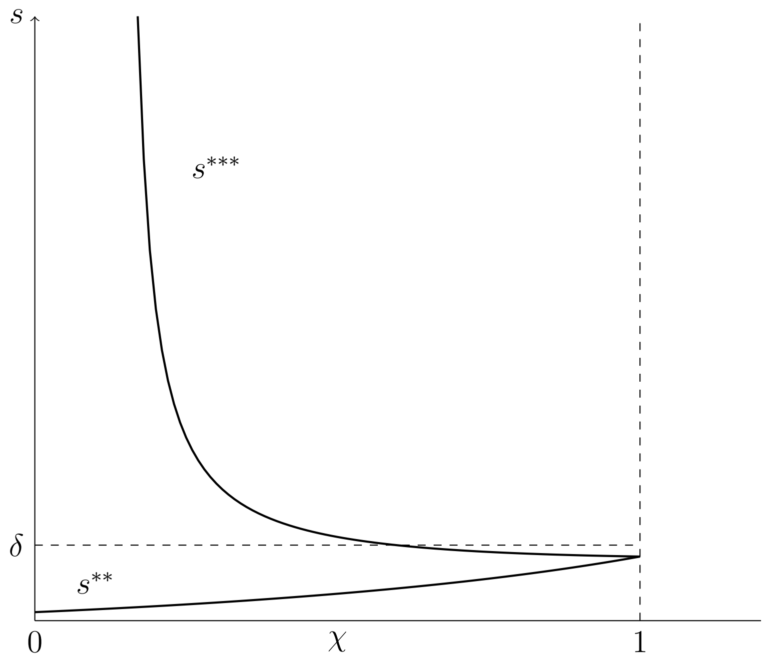

Figure 5: s^{**} and s^{***} as a function of χ

off in the low tax equilibrium. If, on the other hand, $(1-\underline{\tau})r(\underline{\lambda})-(1-\overline{\tau})r(\overline{\lambda})-(\overline{\tau}-\underline{\tau})q<0$, the condition for the likely rich to be worse off in the low tax equilibrium reads:

$$s > \frac{\delta(\overline{\tau}-\underline{\tau})(1-q)}{(\overline{\tau}-\underline{\tau})q+(1-\overline{\tau})r(\overline{\lambda})-(1-\underline{\tau})r(\underline{\lambda})} \equiv s^{***}(\chi). \tag{31}$$

Obviously, whether the likely rich are worse off in the low tax equilibrium is an interesting question only when the low tax equilibrium is possible. Figure 5 depicts s^{**} and s^{***} as a functions of χ. As we have seen, the low tax equilibrium occurs if $s > s^{**}$. By definition of s^{***}, the rich are worse off in the low tax equilibrium if $s > s^{***}$. For $\chi = 1$ the thresholds s^{**} and s^{***} coincide. Therefore, only for the fully Bayesian agents the optimism of the likely poor necessarily makes the rich worse off. For $\chi < 1$ this is not necessarily the case.

Proposition 8 (**The welfare of the likely rich**). *Whether the likely rich are worse off in the low tax equilibrium depends on the degree of the Bayesian sophistication and the value of anticipation.*

(i) For $\chi = 1$, the likely rich are worse off in the low tax equilibrium than in the high tax equilibrium.

(ii) For $\chi < 1$, the likely rich are worse off in the low tax equilibrium only if $s > s^{***}(\chi)$.

In Figure 5, below the lower curve, the POUM effect does not occur. Between the two curves, the POUM effect occurs and it makes the likely rich better off. Above the upper curve, the POUM effect occurs, and it makes the likely rich worse off.

Interestingly, an implication of the model is that the fully Bayesian likely rich are worse off with low taxes if the value of anticipation is high enough for likely poor to choose optimism and low taxes. Again, however, completely sophisticated cognitive technology might be of only theoretical interest. The threshold value s^{***} goes up fairly rapidly for $\chi < 1$, which makes this result less relevant.

4.3 Sincere Voting

The beliefs are most likely to be distorted by desires if the individual cost of holding biased beliefs is small, as is the case in voting if the probability of being pivotal is very small (Bénabou & Tirole, 2016). An alternative assumption about the voting behavior of agents is that they do not consider themselves to be pivotal in the determination of the tax policy and, therefore, form their beliefs without taking into account how it affects their policy preferences and voting.

In the model of the current work, agents trade their optimism against redistribution. If we let the agents ignore this trade-off by assuming sincere voting, the only thing restricting the optimism of agents are the constraints of the cognitive technology. Therefore, taking τ^* as given, the dominating action for the likely poor is to choose $\lambda = 0$ for all $s > 0$: The lower λ they choose, the higher anticipatory utility they can expect. The loss of income and consumption in period 2 due to less redistribution does not enter the trade-off since the agents do not think they can in any way influence the policy outcome. In the unique equilibrium all agents recall $\hat{\sigma} = y_H$, they expect at least average income, and the tax policy is $\tau^* = \underline{\tau}$. This is curiously the equilibrium even if the likely poor do not value anticipation very much and are worse off in the equilibrium than if they had all been realists and voted for high taxes.

Interestingly, another way to motivate sincere voting is to derive it as a limiting case of our benchmark model. When the range of the feasible tax rates goes to zero, the threshold s^{**} goes to zero as well: $\overline{\tau} - \underline{\tau} = 0$ implies $s^{**}(\chi) = 0$, and choosing $\lambda = \underline{\lambda}$ over $\lambda = \overline{\lambda}$ is optimal for all $s > 0$. When the upper and lower bounds of the tax policy coincide, the likely poor cannot affect the tax rate by voting, and it is optimal for them to indulge in optimism.

For the clarity of exposition, we consider the case $\tau \in [0,1]$.[28] The likely poor take τ as given and choose λ to maximize

$$\begin{aligned} U_{0,i}(\lambda) &\equiv \lambda u_{0,i}(y_L, y_L, \tau) + (1-\lambda) u_{0,i}(y_L, y_H, \tau) \\ &= (1-\lambda)\left[\delta s[(1-\tau)[r(\lambda)y_H + (1-r(\lambda))y_L] + \tau\bar{y}] + \delta^2[(1-\tau)y_L + \tau\bar{y}]\right] \\ &+ \lambda\left[(\delta s + \delta^2)[(1-\tau)y_L + \tau\bar{y}]\right]. \end{aligned} \tag{32}$$

The best response, independently of the choices of others, is $\lambda = 0$. This implies an equilibrium tax rate of $\tau^* = 0$.

Proposition 9 (Politico-economic equilibrium, sincere voting). *If the likely poor do not condition their belief and voting choices on the tax policy, then, for all $s > 0$, there is a unique equilibrium, where $\lambda^* = 0$, the likely rich recall $\hat{\sigma}_i = y_H$, and $\tau^* = 0$.*

The utility of a representative likely poor agent is

$$U_{0,i}(0) = \delta s[r(0)y_H + (1-r(0))y_L] + \delta^2 y_L, \tag{33}$$

whereas if the likely poor would have coordinated choosing $\lambda \in [\frac{1}{2(1-q)}, 1]$, a representative likely poor agent would have enjoyed utility

$$U_{0,i}(\lambda) = \delta s\bar{y} + \delta^2\bar{y} \quad \forall \lambda \in [\frac{1}{2(1-q)}, 1]. \tag{34}$$

From Lemma 1, we know that if $s < s^*$, (34) is greater than (33).

Proposition 10 (Welfare of the likely poor). *If $s < s^*$, the likely poor are worse off in the low tax equilibrium, than if they had coordinated on voting for high taxes.*

A free-riding problem emerges among the likely poor: for each, it is individually rational to indulge in optimism, but with coordinated actions they could increase their payoffs. This case is similar to the public goods game, where the individually rational agents do not contribute even if they would all be better off by contributing. Here the public good is the redistribution, and the cost of contribution is lower anticipatory utility. However, the likely poor coordinating on realism to support high taxes is not necessarily a Pareto improvement when considering the whole electorate, as providing a public good in

[28] The case $\tau \in [\underline{\tau}, \overline{\tau}]$ is similar.

a public good game is. As seen in section 4.2, the likely rich are worse off in the high tax equilibrium if $s < s^{***}$ and $\chi < 1$. However, if $\chi = 1$, then unique equilibrium is Pareto-inferior and the likely poor coordinating on realism would be a Pareto improvement.

In contrast to the case of strategic belief formation, which admittedly is a strong requirement on the behavior of the voters, sincere voting always leads to the POUM effect. When the likely poor do not think that their own beliefs will influence the policy outcome, they maximize their utility by maximizing their optimism.

5 Conclusion

Over-optimism seems to be an important mechanism for the POUM hypothesis. We have formalized this mechanism by modeling the means and reasons for belief distortion and derived the conditions in which the poor majority of voters distort their beliefs enough to prefer low taxes in the time of voting. The poor do not expropriate the rich because they themselves believe to be rich someday, and they value these beliefs.

These motivated prospects of upward mobility emerge endogenously as a result of agents' choices between anticipation and consumption. The crucial factors in these choices are the value of anticipation and the relative differences in anticipation and consumption between the potential equilibria.

First, the more the likely poor expect to gain in anticipation when forming biased beliefs, the more biased these beliefs will be. Specifically, if the incomes or perceived incomes of the rich increase while transfers stagnate, the poor will be more likely to indulge in optimism and vote for low taxes. Hence, the striking result is that contrary to the benchmark model of Meltzer and Richard (1981), where the increase in inequality always increases the demand for redistribution, in my model, an increase in inequality can decrease the demand for redistribution.

Second, the less the likely poor expect to lose in consumption when forming biased beliefs, the more biased these beliefs will be. How much the likely poor can expect to lose in consumption depends on the potential tax rates in different equilibria. Hence, the smaller is the difference in the potential policy outcomes, the more likely the POUM effect is. Specifically, if the voters do not think that their vote has an impact in determining the policy outcome, that is, if they do not act strategically, they always form the most optimistic beliefs possible and, therefore, vote for low taxes. If the value of anticipation is low, individually and collectively rational choices diverge, and the poor voters are trapped in a bad equilibrium. By coordinating in voting for higher taxes, they could achieve higher welfare. In this case, the likely poor vote against their own self-interest.

The feasibility of the POUM effect also depends crucially on the specification of the cognitive technology, namely, on the naivete parameter χ. The less constraining the cognitive technology is, the more voters can bias their beliefs. Therefore, the POUM effect becomes more feasible as an explanation for the limited size of the government in democracies when we specify the cognitive technology with small values of χ. This can be clearly seen when comparing the results of Minozzi's (2013) POUM model with our results. In Minozzi's model agents are naive and can effectively choose their beliefs without the restrictions of prior beliefs or reality. When making a more conventional assumption about the voters forward-looking behavior and setting $\chi = 1$ corresponding to the standard Bayesian rationality in belief updating, the poor voters cannot bias their beliefs enough for the POUM effect to occur. This result, however, hinges on the simple specification with linear policy preferences and a policy choice between complete equalization and complete laissez-faire. By exogenously restricting the possible tax policies, it is shown that the POUM effect can be an important factor in voting behavior even if we endow the voters with a more realistic cognitive technology than in Minozzi (2013).

References

Akerlof, G. A., & Dickens, W. T. (1982). The economic consequences of cognitive dissonance. *The American economic review*, *72*(3), 307–319.

Alesina, A., & Angeletos, G.-M. (2005). Fairness and redistribution. *American Economic Review*, *95*(4), 960–980.

Alesina, A., Cozzi, G., & Mantovan, N. (2012). The evolution of ideology, fairness and redistribution. *The Economic Journal*, *122*(565), 1244–1261.

Alesina, A., & Giuliano, P. (2010). The power of the family. *Journal of Economic growth*, *15*(2), 93–125.

Alesina, A., & Giuliano, P. (2011). Preferences for redistribution. In *Handbook of social economics* (Vol. 1, pp. 93–131). Elsevier.

Alesina, A., Glaeser, E., & Glaeser, E. L. (2004). *Fighting poverty in the us and europe: A world of difference.* Oxford University Press.

Alesina, A., Glaeser, E., & Sacerdote, B. (2001). *Why doesn't the us have a european-style welfare system?* (Tech. Rep.). National bureau of economic research.

Alesina, A., & La Ferrara, E. (2005). Preferences for redistribution in the land of opportunities. *Journal of public Economics*, *89*(5-6), 897–931.

Alicke, M. D., & Govorun, O. (2005). The better-than-average effect. *The self in social judgment*, *1*, 85–106.

Austen-Smith, D. (2000). Redistributing income under proportional representation. *Journal of Political Economy*, *108*(6), 1235–1269.

Averill, J. R., & Rosenn, M. (1972). Vigilant and nonvigilant coping strategies and psychophysiological stress reactions during the anticipation of electric shock. *Journal of Personality and Social Psychology*, *23*(1), 128.

Bénabou, R. (1996). Inequality and growth. *NBER macroeconomics annual*, *11*, 11–74.

Bénabou, R. (2000). Unequal societies: Income distribution and the social contract. *American Economic Review*, *90*(1), 96–129.

Bénabou, R. (2008). Ideology. *Journal of the European Economic Association*, *6*(2-3), 321–352.

Bénabou, R. (2012). Groupthink: Collective delusions in organizations and markets. *Review of Economic Studies*, *80*(2), 429–462.

Bénabou, R. (2015). The economics of motivated beliefs. *Revue d'économie politique*, *125*(5), 665–685.

Bénabou, R., & Ok, E. A. (2001). Social mobility and the demand for redistribution: the poum hypothesis. *The Quarterly Journal of Economics*, *116*(2), 447–487.

Bénabou, R., & Tirole, J. (2002). Self-confidence and personal motivation. *The Quarterly Journal of Economics*, *117*(3), 871–915.

Bénabou, R., & Tirole, J. (2006). Belief in a just world and redistributive politics. *The Quarterly journal of economics*, *121*(2), 699–746.

Bénabou, R., & Tirole, J. (2016). Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives*, *30*(3), 141–64.

Bernheim, B. D., & Thomadsen, R. (2005). Memory and anticipation. *The Economic Journal*, *115*(503), 271–304.

Black, D. (1948). On the rationale of group decision-making. *Journal of political economy*, *56*(1), 23–34.

Borck, R. (2007). Voting, inequality and redistribution. *Journal of economic surveys*, *21*(1), 90–109.

Braman, E., & Nelson, T. E. (2007). Mechanism of motivated reasoning? analogical perception in discrimination disputes. *American Journal of Political Science*, *51*(4), 940–956.

Brunnermeier, M. K., & Parker, J. A. (2005). Optimal expectations. *American Economic Review*, *95*(4), 1092–1118.

Caplin, A., & Leahy, J. (2001). Psychological expected utility theory and anticipatory feelings. *The Quarterly Journal of Economics*, *116*(1), 55–79.

Checchi, D., & Filippin, A. (2004). An experimental study of the poum hypothesis. In *Inequality, welfare and income distribution: Experimental approaches* (pp. 115–136). Emerald Group Publishing Limited.

Cojocaru, A. (2014). Prospects of upward mobility and preferences for redistribution: Evidence from the life in transition survey. *European Journal of Political Economy*, *34*, 300–314.

Cook, J. O., & Barnes Jr, L. W. (1964). Choice of delay of inevitable shock. *The Journal of Abnormal and Social Psychology*, *68*(6), 669.

Corneo, G., & Grüner, H. P. (2002). Individual preferences for political redistribution. *Journal of public Economics*, *83*(1), 83–107.

Cruces, G., Perez-Truglia, R., & Tetaz, M. (2013). Biased perceptions of income distribution and preferences for redistribution: Evidence from a survey experiment. *Journal of Public Economics*, *98*, 100–112.

Cukierman, A., & Spiegel, Y. (2003). When is the median voter paradigm a reasonable guide for policy choices in a representative democracy? *Economics & Politics*, *15*(3), 247–284.

De Bondt, W. F., & Thaler, R. H. (1995). Financial decision-making in markets and firms: A behavioral perspective. *Handbooks in operations research and management science*, *9*, 385–410.

Dixit, A., & Londregan, J. (1998). Ideology, tactics, and efficiency in redistributive politics. *The Quarterly Journal of Economics*, *113*(2), 497–529.

Downs, A. (1957). An economic theory of political action in a democracy. *Journal of political economy*, *65*(2), 135–150.

Esping-Andersen, G. (1999). *Social foundations of postindustrial economies*. OUP Oxford.

Fischer, J. A. (2009). The welfare effects of social mobility.

Fong, C. (2001). Social preferences, self-interest, and the demand for redistribution. *Journal of Public economics*, *82*(2), 225–246.

Gilens, M. (2005). Inequality and democratic responsiveness. *Public Opinion Quarterly*, *69*(5), 778–796.

Gottschalk, P., & Spolaore, E. (2002). On the evaluation of economic mobility. *The Review of Economic Studies*, *69*(1), 191–208.

Hirschman, A. O., & Rothschild, M. (1973). The changing tolerance for income inequality in the course of economic development: with a mathematical appendix. *The Quarterly Journal of Economics*, *87*(4), 544–566.

Iversen, T., & Soskice, D. (2006). Electoral institutions and the politics of coalitions: Why some democracies redistribute more than others. *American Political Science Review*, *100*(2), 165–181.

Karabarbounis, L. (2011). One dollar, one vote. *The Economic Journal*, *121*(553), 621–651.

Kopczuk, W., & Slemrod, J. (2005). Denial of death and economic behavior. *Advances in Theoretical Economics*, *5*(1).

Köszegi, B. (2010). Utility from anticipation and personal equilibrium. *Economic Theory*, *44*(3), 415–444.

Kunda, Z. (1990). The case for motivated reasoning. *Psychological bulletin*, *108*(3), 480.

Lerman, C., Hughes, C., Lemon, S. J., Main, D., Snyder, C., Durham, C., … Lynch, H. T. (1998). What you don't know can hurt you: adverse psychologic effects in members of brca1-linked and brca2-linked families who decline genetic testing. *Journal of Clinical Oncology*, *16*(5), 1650–1654.

Löwenstein, G. (1987). Anticipation and the valuation of delayed consumption. *The Economic Journal*, *97*(387), 666–684.

Luebker, M. (2014). Income inequality, redistribution, and poverty: Contrasting rational choice and behavioral perspectives. *Review of Income and Wealth*, *60*(1), 133–154.

Lupu, N., & Pontusson, J. (2011). The structure of inequality and the politics of redistribution. *American Political Science Review*, *105*(2), 316–336.

Mahler, V. A. (2008). Electoral turnout and income redistribution by the state: A cross-national analysis of the developed democracies. *European journal of political research*, *47*(2), 161–183.

Meltzer, A. H., & Richard, S. F. (1981). A rational theory of the size of government. *Journal of political Economy*, *89*(5), 914–927.

Minozzi, W. (2013). Endogenous beliefs in models of politics. *American Journal of Political Science*, *57*(3), 566–581.

Moore, D. A., & Healy, P. J. (2008). The trouble with overconfidence. *Psychological review*, *115*(2), 502.

OECD. (2018). *Tax revenue (indicator).* doi: 10.1787/d98b8cf5-en. ((Accessed on 07 April 2018))

Okun, A. M. (2015). *Equality and efficiency: The big tradeoff.* Brookings Institution Press.

Piketty, T. (1995). Social mobility and redistributive politics. *The Quarterly journal of economics*, *110*(3), 551–584.

Rabin, M. (2002). A perspective on psychology and economics. *European economic review*, *46*(4-5), 657–685.

Ravallion, M., & Lokshin, M. (2000). Who wants to redistribute?: The tunnel effect in 1990s russia. *Journal of public Economics*, *76*(1), 87–104.

Redlawsk, D. P. (2002). Hot cognition or cool consideration? testing the effects of motivated reasoning on political decision making. *The Journal of Politics*, *64*(4), 1021–1044.

Romer, T. (1974). Individual welfare, majority voting, and the properties of a linear income tax.

Schelling, T. C. (1987). The mind as a consuming organ. *The multiple self*, 177–96.

Skala, D. (2008). Overconfidence in psychology and finance-an interdisciplinary literature review.

Solt, F. (2008). Economic inequality and democratic political engagement. *American Journal of Political Science*, *52*(1), 48–60.

Taber, C. S., & Lodge, M. (2006). Motivated skepticism in the evaluation of political beliefs. *American Journal of Political Science*, *50*(3), 755–769.

Tirole, J. (2002). Rational irrationality: Some economics of self-management. *European Economic Review*, *46*(4-5), 633–655.

Todd, E. (1985). *Explanation of ideology: Family structures and social systems (family, sexuality, and social relations in past times).* Blackwell Oxford.

Weinberg, B. A. (2009). A model of overconfidence. *Pacific Economic Review*, *14*(4), 502–515.

Weinstein, N. D. (1980). Unrealistic optimism about future life events. *Journal of personality and social psychology*, *39*(5), 806.

Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1 . Solve first the optimal recall rate given the constraint $\lambda < \frac{1}{2(1-q)}$. Note that here we are looking for the argument of the maximum in a right-open set. However, as we will see, the argument of the maximum is the lower and closed bound of the set and, hence, the maximum exists.

$$\begin{aligned}
\underline{\lambda} &= \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{\lambda[\delta s y_L + \delta^2 y_L] + (1-\lambda)[\delta s[r(\lambda) y_H + (1-r(\lambda)) y_L] + \delta^2 y_L]\right\} \\
&= \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \{(1-\lambda) r(\lambda)\} \\
&= \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{\frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)}\right\} \qquad (35)
\end{aligned}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right) = \frac{[\chi(1-q)-q]^2 - [\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2} < 0 \qquad (36)$$

and is always negative, since $[\chi(1-q)-q]^2 < [\chi(1-q)]^2$. The optimal recall rate is therefore the lower bound of the constraint, that is, $\underline{\lambda} = 0$.

The utility, given that the agents chooses $\lambda < \frac{1}{2(1-q)}$, in (12) is independent of the choice of λ. The best response is the interval $\overline{\lambda} \in [\frac{1}{2(1-q)}, 1]$. Plugging $\underline{\lambda} = 0$ into (14) and solving for s yields (15).

Proof of Lemma 2. If $s > s^*$ the likely poor will choose the awareness rate $\lambda = 0$ and will not want to deviate by Lemma 1. In this equilibrium, no one ever chooses $\hat{\sigma}_i = y_L$, so the information set following this action is on off-equilibrium path and the beliefs in the information set following $\hat{\sigma} = y_L$ can't be defined using Bayer rule or its variations. If we define $p \equiv Pr[\sigma_i = y_H | \hat{\sigma}_i = y_L]$ and require $p \leq q$, we rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on whether the agents are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of λ given in (11) implies $\tau^* = 0$.

If $s < s^*$, the likely poor choose the awareness rate $\lambda \in [\frac{1}{2(1-q)}, 1]$ and will not want to deviate by Lemma 1. Given the strategies of the likely poor and the likely rich, the belief in the information set following $\hat{\sigma} = y_L$ is $Pr[\sigma = y_H | \hat{\sigma} = y_L] = 1$. Therefore, by deviating, a likely rich agent would end up believing to be likely poor and lose anticipatory utility. Hence, the likely rich have no incentive to deviate. The policy outcome as function of λ given in (11) in this case implies $\tau^* = 1$.

Proof of Proposition 1. By Lemma 2 there is an equilibrium with low taxes if $U^{\underline{\lambda}}_{0,i} - U^{\overline{\lambda}}_{0,i} > 0 \iff s > s^*$.

Proof of Lemma 3. Solve first the optimal recall rate given the constraint $\lambda < \frac{1}{2(1-q)}$. Note that here we are looking for the argument of the maximum in a right-open set. However, as we will see, the argument of the maximum is the lower and closed bound of the set and, hence, the maximum exists.

$$\begin{aligned}
\underline{\lambda} = & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \Big\{ \lambda \left[\delta s[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] \right] \\
& + (1-\lambda) \left[\delta s[(1-\underline{\tau})[r(\lambda)y_H + (1-r(\lambda))y_L] + \delta^2[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] \right] \Big\} \\
= & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{ (1-\lambda) r(\lambda) \right\} \\
= & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\}
\end{aligned} \tag{37}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right) = \frac{[\chi(1-q) - q]^2 - [\chi(1-q)]^2}{[q + \chi(1-q)(1-\lambda)]^2} < 0 \tag{38}$$

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\underline{\lambda} = 0$.

Solve the optimal recall rate given the constraint $\lambda \geq \frac{1}{2(1-q)}$.

$$\begin{aligned}
\overline{\lambda} = & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \Big\{ \lambda \left[\delta s[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] \right] \\
& + (1-\lambda) \left[\delta s[(1-\overline{\tau})[r(\lambda)y_H + (1-r(\lambda))y_L] + \delta^2[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] \right] \Big\} \\
= & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \left\{ (1-\lambda) r(\lambda) \right\} \\
= & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\}
\end{aligned} \tag{39}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right)=\frac{[\chi(1-q)-q]^2-[\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2}<0 \tag{40}$$

and as before, is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\overline{\lambda}=\frac{1}{2(1-q)}$. Plugging in the optimal recall rates $\underline{\lambda}$ and $\overline{\lambda}$ and solving for s yields (23).

Proof of Lemma 4. If $s>s^{**}$ the likely poor will choose the awareness rate $\lambda=0$ and will not want to deviate by Lemma 3. In this equilibrium, no one ever chooses $\hat{\sigma}_i=y_L$, so the information set following this action is on off-equilibrium path and the beliefs in the information set following $\hat{\sigma}_i=y_L$ can't be defined using Bayes rule or the variation of the Bayes rule presented in this work. If we define $p\equiv Pr[\sigma_i=y_H|\hat{\sigma}_i=y_L]$ and require $p\leq q$, we rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on whether the agents are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of λ given in (11) implies $\tau^*=\underline{\tau}$.

If $s<s^{**}$, the likely poor choose the awareness rate $\lambda=\frac{1}{2(1-q)}$ and will not want to deviate by Lemma 3. Given the strategies of the likely poor and the likely rich, the belief in the information set following $\hat{\sigma}=y_L$ is $Pr[\sigma=y_H|\hat{\sigma}=y_L]=1$. Therefore, by deviating, a likely rich agent would end up believing to be likely poor and lose anticipatory utility. The likely rich have no incentive to deviate. The policy outcome as a function of λ given in (11) in this case implies $\tau^*=\overline{\tau}$.

Proof of Proposition 2. By Lemma 4, there is an equilibrium with low taxes if $U_{0,i}^{\underline{\lambda}}-U_{0,i}^{\overline{\lambda}}>0\iff s>s^{**}$.

Proof of Proposition 3.

$$\frac{\partial s^{**}}{\partial\chi}=\frac{-\delta(\overline{\tau}-\underline{\tau})q\big[(1-\underline{\tau})\frac{\partial r(0)}{\partial\chi}-(1-\overline{\tau})(1-\overline{\lambda})\frac{\partial r(\overline{\lambda})}{\partial\chi}\big]}{[(1-\underline{\tau})r(0)-(1-\overline{\tau})(1-\overline{\lambda})r(\overline{\lambda})-(\overline{\tau}-\underline{\tau})q]^2}, \tag{41}$$

where

$$\begin{aligned}
&(1-\underline{\tau})\frac{\partial r(0)}{\partial \chi}-(1-\overline{\tau})(1-\overline{\lambda})\frac{\partial r(\overline{\lambda})}{\partial \chi} \\
&=(1-\overline{\tau})\frac{q(1-q)(1-\overline{\lambda})^2}{[q+\chi(1-q)(1-\overline{\lambda})]^2}-(1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2}<0
\end{aligned} \tag{42}$$

since

$$\begin{aligned}
&(1-\overline{\tau})\frac{q(1-q)(1-\overline{\lambda})^2}{[q+\chi(1-q)(1-\overline{\lambda})]^2}<(1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} \\
&\iff (1-\overline{\tau})[q^2(1-\overline{\lambda})^2+2q\chi(1-q)(1-\overline{\lambda})^2+\chi(1-q)^2(1-\overline{\lambda})^2] \\
&<(1-\underline{\tau})[q^2+2\chi q(1-q)(1-\overline{\lambda})+\chi^2(1-q)^2(1-\overline{\lambda})^2]
\end{aligned} \tag{43}$$

which holds since

$$q^2(1-\overline{\lambda})^2+2q\chi(1-q)(1-\overline{\lambda})^2<q^2+2\chi q(1-q)(1-\overline{\lambda}) \tag{44}$$

and $1-\overline{\tau}<1-\underline{\tau}$. Therefore $\frac{\partial s^{**}}{\partial \chi}>0$.

Proof of Proposition 4.

$$\frac{\partial s^{**}}{\partial \overline{\tau}}=\frac{\delta\Delta y(\bar{y}-y_L)\big[(1-\underline{\tau})[r(\underline{\lambda})-(1-\overline{\lambda})r(\overline{\lambda})]\big]}{[(1-\underline{\tau})r(0)-(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})-(\overline{\tau}-\underline{\tau})q]^2(\Delta y)^2} \tag{45}$$

where

$$r(\underline{\lambda})-(1-\overline{\lambda})r(\overline{\lambda})=\frac{q^2}{[q+\chi(1-q)]2(1-q)[q+\chi\frac{1}{2}(1-2q)]}>0. \tag{46}$$

Therefore $\frac{\partial s^{**}}{\partial \overline{\tau}}>0$.

Proof of Proposition 5.

$$\frac{\partial s^{**}}{\partial \underline{\tau}}=-\frac{\delta\Delta y(\bar{y}-y_L)\big[(1-\overline{\tau})[r(\underline{\lambda})-(1-\overline{\lambda})r(\overline{\lambda})]\big]}{[(1-\underline{\tau})r(0)-(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})-(\overline{\tau}-\underline{\tau})q]^2(\Delta y)^2} \tag{47}$$

where

$$r(\underline{\lambda})-(1-\overline{\lambda})r(\overline{\lambda})=\frac{q^2}{[q+\chi(1-q)]2(1-q)[q+\chi\frac{1}{2}(1-2q)]}>0. \tag{48}$$

Therefore $\frac{\partial s^{**}}{\partial \underline{\tau}}<0$.

We establish a result that is useful in determining the sign of the partial derivatives of s^{**}.

Lemma 5. $(1-\underline{\tau})r(0)-(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})>0$

Proof of Lemma 5.

$$
\begin{aligned}
&(1-\underline{\tau})r(0)-(1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) \\
&=\frac{[2(1-q)(q+\chi\frac{1}{2}(1-2q))(1-\underline{\tau})-(q+\chi(1-q))(1-2q)(1-\overline{\tau})]q}{(q+\chi(1-q))(2(1-q)(q+\chi\frac{1}{2}(1-2q))}.
\end{aligned}
\tag{49}
$$

Define

$$
a\equiv 2(1-q)(q+\chi\frac{1}{2}(1-2q)), \tag{50}
$$

$$
b\equiv q+\chi(1-q))(1-2q), \tag{51}
$$

and write the numerator of (49) as

$$
\begin{aligned}
&[a(1-\underline{\tau})-b(1-\overline{\tau}]q \\
\iff &[a-b-(a\underline{\tau}-b\overline{\tau})]q.
\end{aligned}
\tag{52}
$$

The numerator of (49) are positive if

$$
\begin{aligned}
&a\underline{\tau}-b\overline{\tau}>a-b \\
\iff &a(1-\underline{\tau})>b(1-\overline{\tau})
\end{aligned}
\tag{53}
$$

which holds since $a-b=q>0$ and $\overline{\tau}>\underline{\tau}$ implies $1-\underline{\tau}>1-\overline{\tau}$. The denominator of (49) is positive for all $q\in[0,\frac{1}{2}]$. Since both the denominator and the numerator of (49) are positive, the expression is positive and this establishes the result.

Proof of Proposition 6. Write s^{**} as.

$$
s^{**}=\frac{\delta(\overline{\tau}-\underline{\tau})(\bar{y}-y_L)}{\iota_{net}(\underline{\lambda},\underline{\tau})-\iota_{net}(\overline{\lambda},\overline{\tau})} \tag{54}
$$

where

$$
\iota_{net}(\lambda,\tau):=\lambda[(1-\tau)y_L+\tau\bar{y}]+(1-\lambda)[(1-\tau)(r(\lambda)y_H+(1-r(\lambda))y_L)+\tau\bar{y}] \tag{55}
$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given

λ and τ and $\underline{\lambda} = 0$, and $\overline{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to y_H holding the average income $\bar{y}$ constant.

$$\frac{\partial s^{**}}{\partial y_H} = -\frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)[(1 - \underline{\tau})r(0) - (1 - \overline{\lambda})(1 - \overline{\tau})r(\overline{\lambda})]}{[(1 - \underline{\tau})r(0) - (1 - \overline{\lambda})(1 - \overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} < 0. \tag{56}$$

By lemma 5, the derivative is negative for all parameter values, which implies that s^{**} decreases in y_H, when $\bar{y}$ is hold constant.[29] Compute the partial derivative with respect to y_L holding the average income $\bar{y}$ constant.

$$\frac{\partial s^{**}}{\partial y_L} = -\frac{\delta(\overline{\tau} - \underline{\tau})(y_H - \bar{y})[(1 - \underline{\tau})r(0) - (1 - \overline{\lambda})(1 - \overline{\tau})r(\overline{\lambda})]}{[(1 - \underline{\tau})r(0) - (1 - \overline{\lambda})(1 - \overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} < 0 \tag{57}$$

By lemma 5, the derivative is negative for all parameter values, which implies that s^{**} decreases in y_L, when $\bar{y}$ is hold constant. [30]

Proof of Proposition 7. Write s^{**} as.

$$s^{**} = \frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)}{\iota_{net}(\underline{\lambda}, \underline{\tau}) - \iota_{net}(\overline{\lambda}, \overline{\tau})} \tag{58}$$

where

$$\iota_{net}(\lambda, \tau) := \lambda[(1 - \tau)y_L + \tau\bar{y}] + (1 - \lambda)[(1 - \tau)(r(\lambda)y_H + (1 - r(\lambda))y_L) + \tau\bar{y}] \tag{59}$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given λ and τ and $\underline{\lambda} = 0$, and $\overline{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to $\bar{y}$ holding the average income y_L and y_H constant.

$$\frac{\partial s^{**}}{\partial \bar{y}} = \frac{\delta(\overline{\tau} - \underline{\tau})[(1 - \underline{\tau})r(0) - (1 - \overline{\lambda})(1 - \overline{\tau})r(\overline{\lambda})(y_H - y_L)]}{[(1 - \underline{\tau})r(0) - (1 - \overline{\lambda})(1 - \overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} > 0 \tag{60}$$

By lemma 5, the derivative is positive for all parameter values, which implies that s^{**} increases in $\bar{y}$, when y_L and y_H are hold constant.[31]

---

[29] By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial y_H} = -\frac{\bar{y} - y_L}{(y_H - \bar{y})^2}$, which is the result in Minozzi (2013).

[30] By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial y_L} = -\frac{1}{(y_H - \bar{y})}$, which is the result in Minozzi (2013).

[31] By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^{**}}{\partial \bar{y}} = \frac{y_H - y_L}{(y_H - \bar{y})^2}$, which is the result in Minozzi (2013).

Proof of Proposition 8. First, consider part (i). If $s^{**} \geq s^{***}$, then always when there is a low tax equilibrium, the likely rich are worse off in it. So the condition for the low tax equilibrium implies that the likely rich are worse off in the low tax equilibrium if and only if $s^{**}(\chi) \geq s^{***}(\chi)$. Now, it is easy to see that this condition is satisfied for $\chi = 1$ since $s^{**}(1) = s^{***}(1)$. This establishes part (i). Consider now part (ii). Show first that $s^{**}(\chi) < s^{***}(\chi)\ \forall \chi \in [0, 1)$.

$$
\begin{aligned}
& s^{**}(\chi) < s^{***}(\chi) \\
\iff & -(1-\underline{\tau})r(0) + \frac{1}{2}(1-\overline{\tau})r\left(\frac{1}{2(1-q)}\right) + q(\overline{\tau}-\underline{\tau}) < 0.
\end{aligned} \tag{61}
$$

Now show that the left-hand side of (61) is increasing in χ. The derivative of the left-hand side of (61) with respect to χ is

$$
\begin{aligned}
&(1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} - \frac{1}{4}(1-\overline{\tau})\frac{q(1-2q)}{[q+\frac{1}{2}\chi(1-2q)]^2} \\
>&(1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} - \frac{1}{4}(1-\underline{\tau})\frac{q(1-q)}{[q+\frac{1}{2}\chi(1-2q)]^2} \\
=&(1-\underline{\tau})q(1-q)\left[\frac{1}{[q+\chi(1-q)]^2} - \frac{1}{[2q+\chi(1-2q)]^2}\right],
\end{aligned} \tag{62}
$$

where (62) is positive for all $\chi \in [0, 1)$ since

$$
\begin{aligned}
& [q+\chi(1-q)]^2 < [2q+\chi(1-2q)]^2 \\
\iff & [q+\chi(1-q)]^2 < [q+\chi(1-q)+(1-\chi)q]^2
\end{aligned} \tag{63}
$$

for all $\chi \in [0, 1)$. Hence, the derivative of the left-hand side of (61) with respect to χ is positive and the left-hand side of (61) is increasing in χ. Since $s^{**}(1) = s^{***}(1)$, the left-hand side of (61) is zero when $\chi = 1$. Since the left-hand side of (61) is increasing in χ, it has to be negative for $\chi \in [0, 1)$. This establishes that $s^{**}(\chi) < s^{***}(\chi)\ \forall \chi \in [0, 1)$.

Now since $s^{**}(\chi) < s^{***}(\chi)\ \forall \chi \in [0, 1)$, an existence of a low tax equilibrium does not necessarily mean that $s > s^{***}$ and the likely rich are worse off only if $s > s^{***}$. This establishes part (ii).

Proof of Proposition 9. Denote the optimal choice of the likely poor by λ^*.

$$\begin{aligned}
\lambda^* = \underset{\lambda\in[0,1]}{\arg\max} \Big\{ &(1-\lambda)\left[\delta s[(1-\tau)[r(\lambda)y_H + (1-r(\lambda))y_L] + \tau\bar{y}] + \delta^2[(1-\tau)y_L + \tau\bar{y}]\right] \\
&+ \lambda\left[(\delta s + \delta^2)[(1-\tau)y_L + \tau\bar{y}]\right] \Big\} \\
= \underset{\lambda\in[0,1]}{\arg\max} &\left\{(1-\lambda)r(\lambda)\right\} \\
= \underset{\lambda\in[0,1]}{\arg\max} &\left\{\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right\}
\end{aligned} \tag{64}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q+\chi(1-q)(1-\lambda)}\right) = \frac{[\chi(1-q)-q]^2 - [\chi(1-q)]^2}{[q+\chi(1-q)(1-\lambda)]^2} < 0 \tag{65}$$

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\lambda^* = 0$. Since the maximum is unique, the choice $\lambda = \lambda^*$ stictly dominates all other choices of λ and, hence, the unique equilibrium is all the likely poor choosing λ^*.

Proof of Proposition 10. By Lemma 1, if $s < s^*$, (34) is greater than (33).